Instructions: This is a sample exam for Exam #1 in Math 314. Like the actual exam, it consists of five problems. Do all five, showing your work and *explaining your assertions*. Each problem is worth 10 points, for a total of 50 points. Partial credit is given as appropriate on the exam.

Note: Extra credit will be given to those who hand in this sample exam to their TA by 3pm on Friday, Feb. 15, or who submit it in lecture on Feb. 15.

1. Let $\mathcal{P}_2$ be the real vector space consisting of real polynomials $ax^2 + bx + c$ of degree at most two. Let W be the subset of $\mathcal{P}_2$ consisting of the polynomials $f(x) \in \mathcal{P}_2$ such that $f'(0) = 0$. Is W a subspace of $\mathcal{P}_2$? If it is, find a basis for W, and determine the dimension of W.

2. For each of the following, does the map define a linear transformation $\mathbb{R}^2 \to \mathbb{R}^2$. If so, find the kernel (nullspace) and image (range).
   a) $(x, y) \mapsto (x^2, y^2)$.
   b) $(x, y) \mapsto (x + y, 0)$.

3. Suppose that c is a real number. Find the reduced row echelon form of the matrix

$$A = \begin{pmatrix} 1 & 3 & 1 \\ 2 & 7 & 3 \\ 1 & 5 & c \end{pmatrix}$$

and determine if A is invertible. Your answer should depend upon the value of c.

4. Suppose that A is an $n \times n$ matrix with the property that $A^2 = 0$. Show that A is *not* invertible, but that $I + A$ *is* invertible.

5. Define a multiplication law on $\mathbb{R}^2$ by setting

$$(a, b) \cdot (c, d) = (ac, bd).$$

Is $\mathbb{R}^2$ a field, with respect to the usual vector addition law and the above multiplication law? (Hint: What is the multiplicative identity? Which elements have a multiplicative inverse?)