Instructions: These are additional study problems for Exam #3. To practice for the exam, do all of these problems, showing your work and explaining your assertions. Allow yourself 10 minutes per problem. These problems supplement the problems on Sample Exam #3.

1. Prove that the real matrices $\begin{pmatrix} 2 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 6 \end{pmatrix}$ and $\begin{pmatrix} 3 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 5 \end{pmatrix}$ are not similar.

2. a) Find the eigenvalues of $A = \begin{pmatrix} 3 & 4 \\ 4 & -3 \end{pmatrix}$.

b) Show that there is a real matrix B such that $B^5 = A$.

3. Find all diagonal matrices that are similar to $A = \begin{pmatrix} 3 & 1 & 0 \\ 1 & 3 & 0 \\ -1 & -1 & 2 \end{pmatrix}$. Explain.

4. Let A be an $n \times n$ matrix. Show that A is not invertible if and only if 0 is an eigenvalue of A.

5. Explicitly find all 2×2 diagonalizable matrices over $\mathbb{C}$ whose only eigenvalue is 2. Justify your assertion.

6. With respect to the usual dot product on $\mathbb{R}^3$, find an orthogonal basis for the subspace W of $\mathbb{R}^3$ that is spanned by the two vectors $(1, 1, 1)$ and $(1, 2, 3)$. Also find a basis for the orthogonal complement $W^\perp$ of W.

7. a) Is there an orthonormal basis of $\mathbb{R}^3$ consisting of eigenvectors of $A = \begin{pmatrix} 1 & 0 & -2 \\ 0 & 3 & 0 \\ 2 & 0 & 1 \end{pmatrix}$?

b) Is there an orthonormal basis of $\mathbb{R}^3$ consisting of eigenvectors of $B = \begin{pmatrix} 1 & 0 & 2 \\ 0 & 3 & 0 \\ 2 & 0 & 1 \end{pmatrix}$?

Explain.