Math 370 Problem Set #4 Due week of Oct. 1, 2007, in lab.

Read Hoffman and Kunze, Chapter 2, Sections 4-6, and Chapter 3, Section 1.

1. From Hoffman and Kunze, Chapter 2, do these problems:
Page 55, #2. Page 66, #3.

2. From Hoffman and Kunze, Chapter 3, do these problems:
Pages 73-74, #1,2,4,5,8,10.

3. Which of the following are linear transformations? For each one that is, find the kernel (nullspace) and the image (range).

a) $T : \mathbb{R}^3 \to \mathbb{R}^4$ sending $(x, y, z) \mapsto (x - y, y - z, z - x, 0)$.

b) $M : \mathbb{R}^2 \to \mathbb{R}$ sending $(x, y) \mapsto xy$.

c) $D : V \to V$ (where V is the vector space of infinitely differentiable functions on $\mathbb{R}$) sending $f \mapsto f'$.

4. Let $S : \mathbb{R}^2 \to \mathbb{R}^3$ be the linear transformation taking $(x, y) \in \mathbb{R}^2$ to $(a, b, c) \in \mathbb{R}^3$ whenever

$$\begin{pmatrix} 1 & 2 \\ 2 & 4 \\ 3 & 6 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} a \\ b \\ c \end{pmatrix}.$$

Find the kernel and image of S, describing them both geometrically and in terms of equations.

5. Let V be a real vector space, and suppose that $S : V \to \mathbb{R}$ and $T : V \to \mathbb{R}$ are linear transformations. Define $P : V \to \mathbb{R}^2$ by $P(v) = (S(v), T(v))$.

a) Show that P is a linear transformation.

b) Find the kernel of P in terms of the kernels of S and T.