Math 370 Problem Set #8 Due week of Oct. 29, 2007, in lab.

Read Hoffman and Kunze, Chapter 3, Section 7, and Chapter 4, Sections 1-3.

1. From Hoffman and Kunze, Chapter 3, do these problems:
Pages 115-116, #1, 7.

2. From Hoffman and Kunze, Chapter 4, do these problems:
Pages 122-123, #1(a), 4-6. Pages 126-127, #5, 6. [Hint for #6: what is $L(1)$? $L(x)$?]

3. Let X, Y be subspaces of a finite dimensional vector space V and assume that $V = X \oplus Y$. Show that $V^* = \mathrm{Ann}(X) \oplus \mathrm{Ann}(Y)$. [Hint: Choose bases for X and Y.]

4. a) Show that if $T : V \to W$ is a surjective linear transformation of finite dimensional vector spaces with kernel N, then $\dim(V/N) = \dim(V) - \dim(N)$. [Hint: Consider $\dim(W)$.]

b) Illustrate this with the example $V = \mathbb{R}^3$, $W = \mathbb{R}$, $T(x, y, z) = x + y + z$.

5. Let $T : V \to W$ and $S : W \to Z$ be linear transformations of finite dimensional vector spaces.

a) Show that $(S \circ T)^t = T^t \circ S^t$, and deduce that if $S \circ T = 0$ then $T^t \circ S^t = 0$. [You can do this either using the linear transformations or the corresponding matrices.]

b) Show that if T is surjective then T^t is injective. [Hint: What is the kernel of T^t?]

c) Show that if T is injective then T^t is surjective. [Hint: Pick a basis B of V, and show that $T(B)$ extends to a basis of W.]