Since there is no Thursday lab this week due to Thanksgiving, those in that lab may submit this assignment either at the Tuesday lab or in class on Monday or Wednesday.

Read Hoffman and Kunze, Chapter 5, Sections 2-4, and Chapter 6, Sections 1-2.

1. From Hoffman and Kunze, Chapter 5, do these problems:
Page 148-149, #3,5. Page 155, #2, 7. Page 162, #1, 2, 4.

2. Let $A = \begin{pmatrix} 1 & 0 & 1 \\ 1 & -1 & 0 \\ 0 & 1 & c \end{pmatrix} \in M_n(F)$ where $c \in F$ is a scalar.

a) Find $A^{-1}$, using row reduction.

b) *Using part* (a), determine for which $c \in F$ there is an inverse for $A$.

c) Compute the determinant of $A$.

d) *Using part* (c), determine for which $c \in F$ there is an inverse for $A$, and compute $A^{-1}$ using the formula for inverses in terms of determinants. Does this agree with your answers to parts (a) and (b)?

3. Compute the determinant of the matrix

$$\begin{pmatrix} 1 & 2 & -1 & 3 \\ 0 & 1 & 8 & 2 \\ 0 & 0 & 4 & 0 \\ 2 & 0 & -5 & 0 \end{pmatrix}$$

and determine if this matrix has an inverse. [Hint: Some rows or columns may be easier than others to expand about.] Does your answer depend on the field of scalars?

4. Let $A$ be a $3 \times 2$ matrix and let $B$ be a $2 \times 3$ matrix. Find $\det(AB)$. Explain your answer, and explain the connection to problem 4 on Problem Set #5.

5. Recall that if $F$ if a field, then $GL_n(F)$ consists of the invertible matrices in $M_n(F)$. Consider the following subsets of $GL_n(F)$: $SL_n(F)$ consists of the matrices with determinant equal to 1. $GL_n^+(F)$ consists of the matrices with positive determinant. $GL_n^-(F)$ consists of the matrices with negative determinant. $O_n(F)$ consists of the matrices $A$ such that $AA^t = I$ (orthogonal matrices). $S_n(F)$ consists of the matrices $A \in GL_n(F)$ such that $A = A^t$ (symmetric invertible matrices).

a) Show that $GL_n(F)$ is a group under matrix multiplication.

b) Determine which of the subsets $SL_n(F)$, $GL_n^+(F)$, $GL_n^-(F)$, $O_n(F)$, $S_n(F)$ of $GL_n(F)$ are groups under matrix multiplication (and hence subgroups of $GL_n(F)$).