Math 370 | Problem Set #14 | Optional problem set.

This assignment is optional, and may be handed in at the review session to be held on Tuesday, Dec. 11, at 6:30pm in DRL 4C8.

Reminder: The final exam will take place on Thursday, Dec. 13, from 9-11am, in DRL A4.

Read Hoffman and Kunze, Chapter 8, Sections 3-5.

1. From Hoffman and Kunze, Chapter 8, do these problems:
Pages 288-289, #1, 2, 4. Page 298, #1. Page 310, #8. Page 317, #1, 9. [Hint for p.317, #9: Consider problem 5 of PS 13.]

2. Let V be an inner vector space.
   a) Show that if W is a subspace of V, then $W \subset (W^\perp)^\perp$.
   b) Show that $W = (W^\perp)^\perp$ if V is finite dimensional. [Hint: If $\dim V = n$ and $\dim W = d$, then what is $\dim W^\perp$? $\dim (W^\perp)^\perp$?]

3. In $\mathbb{R}^4$, let V be the span of $(1,0,1,0)$ and $(1,1,3,1)$.
   a) Find an orthonormal basis of V. [Hint: Gram-Schmidt.]
   b) Find the point on V closest to $(1,2,3,4)$.
   c) Express $(1,2,3,4) = v_1 + v_2$ explicitly, where $v_1 \in V$ and $v_2 \in V^\perp$.
   d) Find an orthonormal basis of $V^\perp$.

4. Use Gram-Schmidt to find an orthonormal basis for the subspace of $\mathbb{C}^3$ spanned by $(1,-1,i)$ and $(i,1,i)$, with respect to the usual Hermitian inner product.

5. Over $\mathbb{R}$, which of the following matrices have an orthonormal basis of eigenvectors? What about over $\mathbb{C}$? Also, which of these matrices are symmetric? orthogonal? Hermitian? unitary?

$$A = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 3 & 4 & 5 \end{pmatrix}, \quad B = \begin{pmatrix} 3/5 & -4/5 \\ 4/5 & 3/5 \end{pmatrix}, \quad C = \begin{pmatrix} 1 & 1 \\ 0 & 2 \end{pmatrix}$$