Math 371 Problem Set #1 Due week of Jan. 21, 2008, in lab.

Read Herstein, Chapter 1.

1. From Herstein, Chapter 1, do these problems:

a) Section 1.1, page 9: #8, 9. In #9, also show that S^* is a ring under these operations, and find the additive and multiplicative identity elements. Is S^* a field?

b) Section 1.2, page 17: #8-10. [Hint for #10: First do this for the *positive* elements of each set. Do that by listing the positive rational numbers, by writing a/b before c/d (each in lowest terms) if $a+b < c+d$, or else if $a+b = c+d$ and $a < c$.]

2. Determine which of the following relations are equivalence relations:

a) Two integers are related if their difference is odd.

b) Two lines in the plane are related if they are parallel.

c) Two lines in space are related if they are co-planar (i.e. there is a plane containing both of them).

d) Two ordered triples are related if some rearrangement of the entries make the triples the same.

3. Find a relation that is symmetric and transitive but is not an equivalence relation.

4. For any set S (not necessarily finite), define its *power set* $\mathcal{P}(S)$ (also denoted by S^* in the text) to be the set whose elements are the subsets of S.

a) If S is a finite set, with n elements, how many elements are in $\mathcal{P}(S)$? Use this to show that there is no surjection from S to $\mathcal{P}(S)$ when S is finite.

b) Now let S be an arbitrary (possibly infinite) set. Show that it remains true that there is no surjection ϕ from S to $\mathcal{P}(S)$. [Hint: If $\phi : S \to \mathcal{P}(S)$, then consider the subset $T = \{s \in S \mid s \notin \phi(s)\} \subset S$ as an element of $\mathcal{P}(S)$. If $s \in S$, and $\phi(s) = T$, is $s \in T$?]

c) Suppose that $A_1, A_2, A_3, \ldots$ are sets whose elements are positive integers. Use part (b) to show that some set of positive integers is missing from this list. [If $\mathbb{N}$ is the set of positive integers, we then say that $\mathcal{P}(\mathbb{N})$ is *uncountable* or *non-denumerable*, to indicate that its elements cannot be listed. Contrast this with Herstein, p.17, #10.]