Math 502 Problem Set #4 Due the week of Oct. 2, 2006, in lab.

Read Artin, Chapter 2, sections 4-6 and 10.

From Artin, do these problems:
Section 2.4 (pp.72-73): 8, 13, 21.
Section 2.5 (pp.73-74): 2.
Section 2.6 (pp.74): 1, 2, 5, 8.

Also do the following problems:

1. Which of the following relations are reflexive? symmetric? transitive? equivalence relations?
i) On $\mathbb{Z}$, $a \sim b$ if $a + b$ is even.
ii) On $\mathbb{Z}$, $a \sim b$ if ab is even.
iii) On $\mathbb{R}$, $a \sim b$ if $ab > 0$.
iv) On $\mathbb{R}^\times$, $a \sim b$ if $ab > 0$.

2. Find the logical flaw in the following argument, which purports to prove the (false) assertion that if a relation is symmetric and transitive then it must be reflexive and hence be an equivalence relation: "If $a \sim b$ then $b \sim a$ by symmetry; and so $a \sim a$ by transitivity. So the relation is reflexive." Also, give an example of a relation that shows that the conclusion of the argument need not be true. [You may use an example from problem 1.]

3. Let G be a group and let H be a subgroup of G. For every left coset L of H in G, let $\phi(L) = \{g \in G \,|\, g^{-1} \in L\}$.
a) Show that $\phi(L)$ is a right coset of H in G.
b) Show that ϕ defines a bijection between the set of left cosets of H in G and the set of right cosets of H in G.