Math 502 Problem Set #5 Due the week of Oct. 9, 2006, in lab.

Read Artin, Chapter 2, sections 7-10.

From Artin, do these problems:

Section 2.7 (pp.74-75): 1, 7.

Section 2.8 (p.75): 3, 8.

Section 2.9 (p.76): 2, 3, 4. Also in problem 4, deduce that an integer is divisible by 9 if and only if the sum of its digits is divisible by 9. Then apply this to the number 12345678910.

Section 2.10 (p.76): 5, 7.

Also do the following problems:

1. Find all integers n such that there is an element of D_7 having order n. [Hint: If a group of order m has an element of order m, what can you say about the group?]

2. Let m and n be relatively prime integers.

a) Using the isomorphism $\mathbb{Z}/mn\mathbb{Z} \xrightarrow{\sim} \mathbb{Z}/m\mathbb{Z} \times \mathbb{Z}/n\mathbb{Z}$ (for such m and n), prove that for any integers a and b there is a unique integer c such that $c \equiv a \pmod{m}$, $c \equiv b \pmod{n}$, and $0 \leq c < mn$. [This is called the Chinese Remainder Theorem.]

b) In the case that $m = 5$, $n = 7$, $a = 3$, and $b = 2$, find c.

3. Let G be a group.

a) Prove that $\mathrm{Inn}(G)$ is a normal subgroup of $\mathrm{Aut}(G)$, and hence there is a quotient group $\mathrm{Aut}(G)/\mathrm{Inn}(G)$. This quotient group is called the *outer automorphism group* of G, and we denote it by $\mathrm{Out}(G)$. Show that $\mathrm{Out}(G)$ is the trivial group (of one element) if and only if every automorphism of G is inner.

b) For $G = C_3$, find the groups $\mathrm{Aut}(G)$, $\mathrm{Inn}(G)$, and $\mathrm{Out}(G)$ (in particular finding their orders). Do the same for $G = S_3$.