Math 502 Problem Set #6 Due the week of Oct. 16, 2006, in lab.

Read Artin, Chapter 3, sections 1-5.

From Artin, do these problems:

Section 3.1 (p.104): 1, 2.
Section 3.2 (pp.104-105): 1, 11.
Section 3.3 (pp.105-106): 2, 10.
Section 3.4 (pp.106-107): 10.
Section 3.5 (p.107): 2.
[Optional, for extra credit] Miscellaneous problems (p.108): 2.

Also do the following problems:

1. a) Let $G = M_2(\mathbb{R})$ under addition. Find a subgroup $H \subset G$ such that for $A, B \in G$,

$$A \equiv B \pmod{H} \Leftrightarrow \text{trace}(A) = \text{trace}(B).$$

b) Let $G = D_6$ (the group of symmetries of a regular hexagon) and let v be a vertex of the hexagon. Find a subgroup $H \subset G$ such that for $\sigma, \tau \in G$,

$$\sigma \equiv \tau \pmod{H} \Leftrightarrow \sigma(v) = \tau(v).$$

2. Find all the finite subgroups of the multiplicative group $\mathbb{C}^\times$. Justify your assertion.

3. Define the *commutator subgroup* G' of a group G to be the subgroup of G generated by $\{aba^{-1}b^{-1} \mid a, b \in G\}$.

a) Find G' if $G = \mathbb{Z}, S_3, D_4$.
b) Show that G' is a normal subgroup of G.
c) Show that a group G is abelian if and only if G' is the trivial group.
d) Let N be a normal subgroup of G. Show that G/N is abelian if and only if $G' \subset N$.

4. Prove that the functions e^x, e^{2x}, e^{3x} are linearly independent in the vector space $V = \{\text{differentiable functions}\}$. [Hint: If not, then differentiate twice.]

5. Find all real numbers a such that the vectors $(a, 1, 0), (1, a, 1), (0, 1, a)$ are linearly independent in $\mathbb{R}^3$.