Math 502 Problem Set #12 Due the week of Nov. 27, 2006, in lab.

Read Artin, Chapter 5, section 9, and Chapter 6, sections 1-6.

From Artin, do these problems:

Section 5.2 (pp.188-189): 15.
Section 5.3 (p.189): 2.
Section 5.4 (pp.189-192): 9, 14.
Section 5.9 (p.195): 7.
Section 6.1 (pp.229-230): 3, 14.
Section 6.2 (p.230): 4.
Section 6.4 (pp.230-231): 2, 15.
Section 6.6 (pp.232-233): 2.

Also do the following problems:

1. a) Find all groups of order 35.

b) Find all groups of order 175.

c) Find all groups of order 34. [Hint: For which n is there an element of order n? For each such n, how many elements can have order n? If g has order 17 and h has order 2, what is hgh^{-1}?]

2. a) Show that $\mathrm{Aut}(\mathbb{Z}/n\mathbb{Z}) \approx (\mathbb{Z}/n\mathbb{Z})^\times$, for any positive integer n. [The left hand side refers to automorphisms as a group.]

b) Let $G_1 = \mathbb{Z}/3\mathbb{Z}$, and for $i \geq 1$ let $G_{i+1} = \mathrm{Aut}(G_i)$. For every positive integer n find G_n, and determine which of these are abelian.

c) Do the same with $G_1 = \mathbb{Z}/8\mathbb{Z}$.