Math 602 Sample Exam Fall 2004

For each of the following, either give an example, or else prove that none exists.

1. A non-abelian group of order 55.
2. A non-abelian group of order 121.
3. A simple group of order 256.
4. A finite group that is solvable but not nilpotent.
5. A non-split short exact sequence of finite groups.
6. A finite abelian group whose automorphism group is non-abelian.
7. A non-trivial proper left ideal in $M_3(\mathbb{F}_5)$ (i.e. 3×3 matrices over $\mathbb{F}_5 = \mathbb{Z}/5\mathbb{Z}$).
8. An integral domain of Krull dimension 3.
9. A maximal ideal in $\mathbb{R}[x, y]/(xy - 2)$.
10. A local ring that is an integral domain but not a field.
11. A commutative ring whose nilradical is unequal to its Jacobson radical.
12. An ideal I in an integral domain R such that I is radical but not prime.
13. A non-zero proper ideal of $\mathbb{Q}[[x]]$ that is not maximal.
14. A matrix in $M_2(\mathbb{Q})$ that is triangularizable over $\mathbb{C}$ but not over $\mathbb{R}$.
15. A non-zero alternating 3-form on $\mathbb{R}^2$.
16. A matrix $S \in M_2(\mathbb{C})$ such that $Tv \cdot w = v \cdot Sw$ for all $v, w \in \mathbb{C}^2$, where $S = \begin{pmatrix} i & 1 \\ 2 & -i \end{pmatrix}$.
17. A 4×4 real orthogonal matrix of rank 3.
18. A non-zero nilpotent matrix in $M_3(\mathbb{R})$.
19. A set of eight linearly independent vectors in $\mathbb{R}^3 \otimes \mathbb{R}^4$.
20. An invertible linear transformation $T : \mathbb{R}^5 \to \mathbb{R}^5$ such that $Tv \perp v$ for all $v \in \mathbb{R}^5$.