1. Show that A_4 is isomorphic to a semi-direct product $C_2^2 \rtimes C_3$.

2. Suppose that $N \triangleleft S_5$.

a) Show that if N contains a transposition (a, b) then $N = S_5$. (Hint: The set of transpositions generates S_5.)

b) Show that if $N \cap A_5 = 1$ and $\sigma \in N$, then either $\sigma = 1$ or else σ is a transposition. (Hint: Show that $\sigma^2 = 1$.)

c) Conclude that $N = 1, A_5$, or S_5.

3. Let $n > 2$. Show that the dihedral group D_n of order $2n$ is isomorphic to a semi-direct product $C_r \rtimes C_s$ if and only if $r = n$ and $s = 2$.

4. Determine which of the following groups are isomorphic: Q, D_4, C_8, C_2^3, $(C_2)^2 \rtimes C_2$, where in the last group the generator of the latter C_2 acts by interchanging the two factors of $(C_2)^2$. (This last group is also written as $C_2 \wr C_2$, and it is a special case of what is called as *wreath product*. In general, $N \wr H$ is the semi-direct product $N^H \rtimes H$, where N^H is the direct product of copies of N indexed by the elements of H, and the elements of H act by permuting the factors of N^H by left multiplication on the indices.)

5. Find all groups of order 66, up to isomorphism. Which are simple? solvable? nilpotent? abelian? cyclic? Which are split extensions (of a non-trivial quotient by a non-trivial subgroup)?