Math 624 Problem Set #6 Due Monday, Nov. 9, 2020

In Hartshorne, read Chapter II, sections 4-5.

1. In Hartshorne, Chapter II, do these problems: 4.1, 4.3, 4.6, 4.9, 5.1 - 5.3, 5.7.

2. Determine whether each of the morphisms in the last problem on Problem Set #5 is proper, using either the definition or the valuative criterion. Relate this to the answers that you gave to the properties listed in that earlier problem, concerning consistency.

3. Let $X = \mathbb{P}^1_k$, where k is a field. Which of the following are sheaves of abelian groups? Which are $\mathcal{O}_X$-modules? For those that are the latter, which ones are generated by their global sections? Which are quasi-coherent? finitely generated? coherent?

i) $\mathcal{F}(U) = \mathbb{Z}$ for every non-empty open set U; $\mathcal{F}(\emptyset) = 0$.
ii) $\mathcal{F} = \mathcal{O}^\times$.
iii) $\mathcal{F}(U) = k$ if $\infty \in U$; otherwise $\mathcal{F}(U) = 0$.
iv) $\mathcal{F}(U) = 0$ if $\infty \in U$; otherwise $\mathcal{F}(U) = \mathcal{O}(U)$.
v) $\mathcal{F}(U) = \{f \in \mathcal{O}(U) \mid f \text{ vanishes on } S \cap U\}$, where $S = \{0, 1, \infty\}$.

4. Check directly that the sheaf $\mathcal{O}(3)$ on $\mathbb{P}^2_k$ satisfies each of the five equivalent conditions discussed in class for being coherent on a Noetherian scheme (i.e. don't rely on the equivalence of these five conditions).