Math 624 Problem Set #7 Due Wednesday, Nov. 18, 2020

In Hartshorne, read Chapter II, sections 5-6.

1. In Hartshorne, Chapter II, do these problems: 5.8, 5.12, 5.13, 6.1, 6.4. (Optional study problems, not to be submitted: 5.17, 5.18.)

2. (a) In the situation of problem 3 on Problem Set 6, determine whether each of the items (i)-(v) is locally free, invertible, and very ample.

(b) For each of the following, redo problem 3 on Problem Set 6, including determining whether these are locally free, invertible, and very ample.

i) $\mathcal{F} = \mathcal{O}(r)$, where $r \in \mathbb{Z}$ is a fixed integer.

ii) $\mathcal{F} = \mathcal{O}(r) \oplus \mathcal{O}(s)$, where $r, s \in \mathbb{Z}$ are fixed integers.

iii) $\mathcal{F}(U) = \{$(not necessarily continuous) functions $\phi : U \to k\}$.

3. Let $\mathcal{F} = \bigoplus_{i=1}^{s} \mathcal{O}(r_i)$ on $\mathbb{P}^n_k$.

a) For which choices of s, r_i is $\mathcal{F}$ locally free? Prove your assertion.

b) For which choices of s, r_i is $\mathcal{F}$ locally free of rank 1? Prove your assertion.

c) For which choices of s, r_i is $\mathcal{F}$ very ample? Prove your assertion.

d) For which choices of s, r_i is there a sheaf $\mathcal{G}$ such that $\mathcal{F} \otimes \mathcal{G} = \mathcal{O}$? For each such choice, find $\mathcal{G}$ explicitly.

4. On each of the following k-schemes, determine which of the listed divisors (if any) are linearly equivalent.

a) $X = \mathbb{P}^2$, with homogeneous coordinates $(x : y : z)$. Divisors $L_x, L_y, L_z, L_x + L_y$, where L_x is the prime divisor where $x = 0$ and similarly for L_y, L_z.

b) $X = \mathbb{A}^2$, with affine coordinates (x, y). Divisors $L_x, L_y, L_x + L_y$.

c) $X = \mathbb{P}^1 \times \mathbb{P}^1$, with bihomogeneous coordinates $(x_0 : x_1; y_0 : y_1)$. Divisors L_{x_0}, L_{x_1}, L_{y_0}, L_{y_1}, $L_{x_0} + L_{y_0}$.

d) $X = \mathbb{A}^1$. Divisors $P_0, P_1, P_{-1}, 2P_0, P_1 + P_{-1}$, where P_c is the point where $x = c$.

e) $X = \mathbb{P}^1$. Divisors $P_0, P_1, P_{-1}, 2P_0, P_1 + P_{-1}$, where P_c is the point $(c : 1)$.