Math 624 Problem Set #3 Due Wednesday, Oct. 16, 2002

Part I:

Read Fulton, Chapter 1, sections 8-10, and Chapter 2, sections 1-4.
Do problems:

1.40 on p.25;
2.11-2.13 on pp.39-40;
2.16 on p.42;
2.18, 2.21 on pp.45-46.

Part II:

Read Hartshorne, Chapter I, section 4.
1. In Chapter I, do problems:

2.14, 2.15, 3.4, 3.6, 3.14-3.16, 4.4, 4.5.

2. (a) Show that the complex affine line is not birationally isomorphic to the curve $X \subset \mathbb{A}^2_{\mathbb{C}}$ given by $x^3 + y^3 = 1$. Do this in steps, by assuming instead that there *is* an isomorphism $\Phi : K(\mathbb{A}^2) \to K(X)$ of function fields, and then proceeding as follows:
(i) There are relatively prime non-zero polynomials $p, q, r \in \mathbf{C}[t]$ such that $p^3 + q^3 - r^3 = 0$ identically.
(ii) For these p, q, r, there is the matrix identity

$$\begin{pmatrix} p & q & r \\ p' & q' & r' \end{pmatrix} \begin{pmatrix} p^2 \\ q^2 \\ -r^2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix},$$

where p', q', r' are the derivatives of p, q, r.
(iii) Hence $qr' - rq'$, $rp' - pr'$, $pq' - qp'$ are respectively divisible by p^2, q^2, r^2.
(iv) By examining the degrees of p, q, r, derive a contradiction.

(b) What if the base field is some other algebraically closed field k, instead of $\mathbb{C}$? Does the conclusion of part (a) remain true for *all* such k?