

Part I:

Read Fulton, Chapter 2, sections 5-11, and Chapter 3, section 1.

Do problems:

- 2.24, 2.25, 2.31 on pp.47-49;
- 2.46 on p.55;
- 2.51 on p.60;
- 2.56 on p.62;
- 3.1-3.4, 3.11 on pp.67-70.

Part II:

Read Hartshorne, Chapter I, section 5.

1. In Chapter I, do problems:

- 3.17, 3.21, 4.6, 4.9, 5.1-5.4.

2. Let $P = (0, 0) \in \mathbb{A}^2$, $L = (y = 0) \subset \mathbb{A}^2$ over \mathbb{C} . Consider the following varieties in \mathbb{A}^2 :

- (i) $y = x^2$; (ii) $y^2 = x^3 - x$; (iii) $y^3 = x^3 - 2x^2$; (iv) $y^2 = x^5$.

For each of these varieties V , do the following:

- (a) Draw the (real points of the) graph of V , and find the singular locus of V .
- (b) Find the tangent cone to V at P . Also find the tangent lines to V at P and their multiplicities. [If $I(V) = (f)$ and the initial form (of lowest degree terms) of f is $\prod \ell_i^{n_i}$ where the ℓ_i are linear polynomials defining distinct lines L_i , then the L_i are the *tangent lines* at the origin and the *multiplicity* of L_i is n_i . The *tangent cone* is the union of the tangent lines, counting them with their multiplicities.]
- (c) Find the tangent space to V at P in \mathbb{A}^2 (i.e. the space defined by the linear terms of the above polynomial f). Also calculate $\dim_{\mathbb{C}}(\mathfrak{m}_P/\mathfrak{m}_P^2)$ directly, where \mathfrak{m}_P is the maximal ideal associated to P . Compare. (Also compare to the space spanned by the tangent lines.)
- (d) Find the intersection multiplicity $(L \cdot V)_P$ of L with V at P , and find $(L \cdot V)$. [Cf. Hartshorne, Exercise I 5.4.] Does this agree with the real picture?

3. For each of the varieties V in problem 2, and with $P = (0, 0)$:

- (a) Determine whether $\mathcal{O}_{V,P}$, $\mathcal{O}_{V,P}^{\text{hol}}$, and $\hat{\mathcal{O}}_{V,P}$ are integral domains.
- (b) Determine the multiplicity $\mu_P(V)$ of P on V . [Recall that the *multiplicity* of $V = Z(f)$ at the origin is the degree of the initial form of f . If $\mu_P(V) = 2$ then P is called a *double point*, and if $\mu_P(V) = 3$ then P is called a *triple point*. If P is a double point with two distinct tangent lines, each of multiplicity 1, then it is called a *node*; while if it has only one tangent line with multiplicity 2 then it is called a *cusp*.]
- (c) Determine all the nodes of V , and all the cusps of V .