Math 624 Problem Set #5 Due Wednesday, Oct. 30, 2002

Part I:

Read Fulton, Chapter 3, sections 2-3.
Do problems:

3.12-3.14 on p.73;
3.17, 3.22 on pp.81-82.

Part II:

Read Hartshorne, Chapter I, sections 6-8.
1. In Chapter I, do problems:

5.5, 5.6, 5.10, 6.1, 6.6, 6.7, 7.1, 7.3.
(Optional: Also do problems 5.7, 6.2, 6.4.)

2. Show that $\mathrm{GL}_n(k) := \{\text{invertible } n \times n \text{ matrices over } k\}$ and $\mathrm{SL}_n(k) := \{\text{matrices in } \mathrm{GL}_n(k) \text{ of determinant } 1\}$ are *group varieties* over k (cf. Hartshone Chapter I, problem 3.21). [Group varieties are also called "algebraic groups". Over the field of complex numbers, algebraic groups are also Lie groups, and in particular $\mathrm{GL}_n(\mathbb{C})$ and $\mathrm{SL}_n(\mathbb{C})$ are Lie groups.] For which n are these groups abelian?

3. Find a smooth projective curve having function field $\mathbb{C}(x)[y]/(x^2 - x^4 - y^2)$. Also, find an irreducible curve in $\mathbb{P}^2$ that has this function field and that has no singularities other than nodes.