Math 624 Problem Set #6 Due Friday, Nov. 8, 2002

Part I:

Read Fulton, Chapter 4.
Do problems:
4.14, 4.16, 4.18 on pp.95-96;
4.24 on p.99.

Part II:

Read Hartshorne, Chapter II, sections 1-2.
1. In Chapter II, do problems 1.14, 1.17, 2.1, 2.8, 2.11.
(Optional: In Chapter I, do problems 7.2, 7.4. In Chapter II, do problems 1.7, 1.8, 1.13, 1.16, 1.18, 1.19, 1.21, 2.3, 2.7, 2.9.)

2. Let $X = \mathbb{P}^1_{\mathbb{C}}$, regarded as a complex analytic space, i.e. with the metric topology and equipped with the structure sheaf $\mathcal{H}$ of holomorphic functions. Let n be a positive integer.

a) Show that there is a morphism of sheaves $\phi : \mathcal{H}^* \to \mathcal{H}^*$ given by $f \mapsto f^n$. For which open sets $U \subset X$ is $\phi(U) : \mathcal{H}^*(U) \to \mathcal{H}^*(U)$ injective? surjective? Find the sheaf kernel and sheaf cokernel of ϕ. Into what short exact sequence does this morphism fit?

b) What changes if X is instead regarded as an algebraic variety, with the Zariski topology and structure sheaf $\mathcal{O}$, and taking $\phi : \mathcal{O}^* \to \mathcal{O}^*$? (Remark: It is possible to recapture the situation in part (a) for algebraic varieties if instead one uses the *étale* topology.)

3. a) Draw $\operatorname{Spec} \mathbb{C}[x,t]$, $\operatorname{Spec} \mathbb{C}[x,t]/(t^2)$, and $\operatorname{Spec} \mathbb{C}[x][[t]]$. In each case draw the following loci, showing and labeling each point where they meet:
$(x), (x-2), (t), (t-1), (t-x), (t-x^2), (t^2-x), (1-xt)$.

b) Do the same for $\operatorname{Spec} \mathbb{Z}[t]$, $\operatorname{Spec} \mathbb{Z}[t]/(t^2)$, and $\operatorname{Spec} \mathbb{Z}[[t]]$, showing the loci
$(2), (5), (t), (t-1), (t-2), (t-4), (t^2+1), (1-2t)$.