

Part I:

Read Fulton, Chapter 5, sections 1-3.

Do problems:

- 4.11 on pp.94-95;
- 5.1, 5.2, 5.3ab, 5.10, 5.11 on pp.105-107;
- 5.17, 5.18 on pp.111-112;
- 5.24 on p.117.

Part II:

1. In Chapter II, do problems 1.8, 1.21, 2.5, 2.10, 2.19.

(Optional: In Chapter II, do problems 1.16, 2.2, 2.12.)

2. a) Let $X = \mathbb{P}_{\mathbb{C}}^1$, regarded as a complex analytic space, equipped with the structure sheaf \mathcal{H} . Consider the morphism $\mathcal{H} \rightarrow \mathcal{H}^*$ given on open sets by $f \mapsto e^f$. Is this a monomorphism of sheaves? An epimorphism of sheaves? What if instead one considers it as a morphism of presheaves?

b) Do the same for the morphism $\mathcal{O} \rightarrow \Omega$ given by $f \mapsto df$, on $\mathbb{P}_{\mathbb{C}}^1$ regarded as an algebraic variety.

3. For each of the following rings R , sketch $X = \text{Spec } R$ and determine whether X is connected, irreducible, reduced, integral: $\mathbb{C}[x]/(x^2 - x)$, $\mathbb{C}[x]/(x^3 - x^2)$, $\mathbb{C}[x, y]/(xy - y)$, $\mathbb{Z}/6$, $\mathbb{Z}/12$, $\mathbb{Z}[x]/(2x)$, $\mathbb{Z}[x]/(x^2)$, $\mathbb{Z}[x]/(4)$.

4. Let R be a ring and let $X = \text{Spec } R$.

a) Show that the nilradical of R (i.e. the intersection of the prime ideals of R) is trivial iff X is reduced.

b) Suppose that the nilradical of R is trivial. Show that the Jacobson radical of R (i.e. the intersection of the maximal ideals of R) is trivial iff $\overline{\text{Max } R} = X$ (where we view $\text{Max } R \subset \text{Spec } R = X$, and take its closure).

c) Give an example of a ring such that either the nilradical or the Jacobson radical is trivial, but not both. For this example, verify directly that the assertions of a) and b) hold.

5. (optional) This exercise concerns the notion of a “flasque” sheaf (also called “flabby” in English). See Hartshorne, II, Exercise 1.16 for the definition.

a) Is \mathcal{O} flasque on the algebraic variety $\mathbb{A}_{\mathbb{C}}^1$?

b) Is \mathcal{H} flasque on the complex analytic space $\mathbb{A}_{\mathbb{C}}^1$?

c) Is the sheaf of C^∞ -functions on the topological space \mathbb{R}^2 flasque?

d) Is the sheaf of (not necessarily continuous) functions $\mathbb{R}^2 \rightarrow \mathbb{R}$ flasque?