Math 624 Problem Set #8 Due Monday, Nov. 25, 2002

Part I:

Read Fulton, Chapter 5, sections 4-6.
Do problems:

5.21 on p.115;
5.25 on p.118;
5.30 on p.122;
5.31, 5.32, 5.34-5.37 on pp.125-127.

Part II:

Read Hartshorne, Chapter II, sections 3-4.
1. In Chapter II, do problems 2.9, 2.13, 3.5, 3.8, 3.10, 4.1.
(Optional: In Chapter II, do problems 2.11, 3.9, 3.14, 4.6.)
(Additional study problems: Chapter II, problems 3.15-3.21, 3.23.)

2. a) Show that if $\alpha : X \to S$ and $\beta : Y \to S$ are any maps of sets, then $\{(x, y) \in X \times Y \mid \alpha(x) = \beta(y) \in S\}$ satisfies the universal mapping property of fibre product.

b) Let $S = \mathbb{A}^2_k$, the x, y-plane over a field k (as a scheme). Let $X = \mathbb{A}^3_k$, with coordinates x, y, z; and let $Y \subset S$ be the closed subscheme defined by $y^2 = x$. Let $\alpha : X \to S$ be the obvious projection, and let $\beta : Y \to S$ be the natural inclusion. Describe the scheme $X \times_S Y$ geometrically, and relate its ring of functions to a corresponding tensor product.

3. For each of the following morphisms ϕ of schemes, determine whether ϕ is of finite type, finite, quasi-finite, proper, surjective, and projective.
(i) ϕ is the morphism corresponding to the endomorphism of $k[x]$ given by $x \mapsto x^3$.
(ii) ϕ is the morphism corresponding to the inclusion of rings $k[x] \hookrightarrow k[x, y]/(y^3 - y - x)$.
(iii) ϕ is the morphism corresponding to $k[x] \hookrightarrow k[x, y, y^{-1}]/(y^3 - y - x)$.
(iv) ϕ is the first projection map $\mathbb{P}^1_k \times_k \mathbb{A}^1_k \to \mathbb{P}^1_k$.
(v) ϕ is the second projection map $\mathbb{P}^1_k \times_k \mathbb{A}^1_k \to \mathbb{A}^1_k$.
(vi) ϕ is the morphism corresponding to the inclusion of rings $k[x] \hookrightarrow k(x)$.