Part I:

Read Fulton, Chapter 6, sections 1-3.
Do problems:

6.11, 6.12 on pp.137;
6.17, 6.19, 6.22-6.25 on pp.142-144.

Part II:

Read Hartshorne, Chapter II, section 5.

1. In Chapter II, do problems 5.2, 5.7, 5.12.
(Optional: In Chapter II, do problems 5.1, 5.3, 5.5, 5.8, 5.13, 5.18.)

2. Check directly that the sheaf $\mathcal{O}(3)$ on $\mathbb{P}^2_k$ satisfies each of the five equivalent conditions for being coherent on a Noetherian scheme (i.e. don't rely on the equivalence of these five conditions).

3. Let $\mathcal{F} = \bigoplus_{i=1}^{s} \mathcal{O}(r_i)$ on $\mathbb{P}^n_k$.

a) For which choices of s, r_i is $\mathcal{F}$ locally free? Prove your assertion.

b) For which choices of s, r_i is $\mathcal{F}$ locally free of rank 1? Prove your assertion.

c) For which choices of s, r_i is there a sheaf $\mathcal{G}$ such that $\mathcal{F} \otimes \mathcal{G} = \mathcal{O}$? For each such choice, find $\mathcal{G}$ explicitly.

4. Let $X = \mathbb{P}^1_k$, where k is a field. Which of the following sheaves $\mathcal{F}$ is an $\mathcal{O}_X$-module? For those that are, which ones are generated by their global sections? Which are quasi-coherent? finitely generated? coherent? invertible? very ample?

i) $\mathcal{F}(U) = \mathbb{Z}$ for every non-empty open set U; $\mathcal{F}(\emptyset) = 0$.
ii) $\mathcal{F} = \mathcal{O}^*$.
iii) $\mathcal{F}(U) = k$ if $\infty \in U$; otherwise $\mathcal{F}(U) = 0$.
iv) $\mathcal{F}(U) = 0$ if $\infty \in U$; otherwise $\mathcal{F}(U) = \mathcal{O}(U)$.
v) $\mathcal{F} = \mathcal{O}(r)$, where $r \in \mathbb{Z}$ is a fixed integer.
vi) $\mathcal{F} = \mathcal{O}(r) \oplus \mathcal{O}(s)$, where $r, s \in \mathbb{Z}$ are fixed integers.
vii) $\mathcal{F}(U) = \{f \in \mathcal{O}(U) \mid f \text{ vanishes on } S \cap U\}$, where $S = \{0, 1, \infty\}$.
viii) $\mathcal{F}(U) = \{(\text{not necessarily continuous) functions } \phi : U \to k\}$.