

Read Hartshorne, Chapter IV, section 4.

Read Fulton, Chapter 8, sections 1-3.

1. In Hartshorne, Chapter IV, do problems 2.1, 3.1. Optional: problem 2.2.
2. In Fulton, Chapter 8, do problems 8.4 (p.191); 8.14 and 8.16 (pp.201-202). Optional: problem 8.12 (p.196); 8.20 and 8.21 (p.202).
3. Recall that if k is an algebraically closed field then a smooth connected projective k -curve X has genus 0 if and only if $X \approx \mathbb{P}_k^1$.
 - a) Show by example that this assertion is false if k is not assumed algebraically closed.
 - b) What goes wrong with the proof in that case? (Recall that Riemann-Roch still holds for non-algebraically closed fields.)
 - c) Show that over a general field k , if X has genus 0, then $X \approx \mathbb{P}_k^1$ iff X has a k -point.
4. Show that if k is algebraically closed of characteristic $p \neq 0$, then there are unramified covers $X \rightarrow \mathbb{A}_k^1$ such that the smooth projective model of X has arbitrarily large genus.
5. Redo PS2 problem 5(c) for the other four regular solids.
6. If $Y \rightarrow X$ is a Galois branched cover of regular connected schemes of dimension 1, with Galois group G , and if $Q \in Y$, define the *decomposition group* $D_Q \subset G$ to consist of the elements $g \in G$ such that $g(Q) = Q$.
 - a) Show that D_Q is a subgroup of G .
 - b) Show that the order of D_Q is equal to the ramification index e_Q if $Y \rightarrow X$ is a branched cover of curves defined over an algebraically closed field k . Is this true for *all* fields k ? Is an inequality true in one direction?
 - c) Let $k = \mathbb{C}$, let $Y = \mathbb{A}^1$ be the y -line, let $X = \mathbb{A}^1$ be the x -line, and consider the cover $Y \rightarrow X$ given by $y^2 = x$. Show that this cover is Galois; find the Galois group; and find the branch locus. Also find D_Q if $Q \in Y$ lies over $x = 0$. Redo this with $x = 1$ and with $x = -1$. What if instead $k = \mathbb{R}$?
 - d) Let $Z \subset \mathbb{A}^3$ be the affine curve given by $y^2 = x, z^2 = x - 1$, and let $\pi : Z \rightarrow X = \mathbb{A}^1$ be the cover given by projection onto the x -coordinate. Redo part (c) for this cover.
 - e) Let $X = \text{Spec } \mathbb{Z}$, let $Y = \text{Spec } \mathbb{Z}[i]$, and let $\pi : Y \rightarrow X$ be the morphism corresponding to the inclusion $\mathbb{Z} \hookrightarrow \mathbb{Z}[i]$. Show that this cover is Galois, and find the Galois group and branch locus. Then find the decomposition group D_Q if $Q \in Y$ lies over the point corresponding to the prime (2). Redo this with the prime (3) and with the prime (5).