

Read Fulton, Chapter 8, section 6.

Read Hartshorne, Chapter IV, section 6; and Chapter III, sections 1-4.

1. In Fulton, Chapter 8, do problems 8.31 (p.213); 8.34 (p.214).
2. a) In Hartshorne, Chapter IV, do problems 4.8, 5.2. Optional: problems 5.1, 6.1.
b) In Hartshorne, Chapter III, do problems 2.1(a), 2.2. Optional: problem 2.1(b).
3. Let X be a smooth connected projective curve of genus g . For any positive integer n , say that X has property D_n if for every choice of distinct points $P_1, \dots, P_n \in X$, there is a rational function on X with poles of order exactly 1 at each P_i , and no other poles. What can you say about the set of n 's such that X satisfies property D_n ?
4. Suppose that $\pi : Y \rightarrow X$ is an étale morphism; i.e. it is locally given on the ring level by adjoining elements y_1, \dots, y_m subject to relations f_1, \dots, f_m such that the relative Jacobian matrix $(\partial f / \partial y)$ is invertible.
 - a) Show that if X is a smooth k -scheme then so is Y .
 - b) Let $\phi : Z \rightarrow X$ be any morphism, and consider the fibre product $Z \times_X Y = \{(z, y) \mid \phi(z) = \pi(y)\}$. Show that the first projection $p_1 : Z \times_X Y \rightarrow Z$ is étale. (Hint: First show that p_1 is defined by the same equations as define π .)
5. Suppose that $\pi : Y \rightarrow X$ is a finite étale cover of smooth connected k -curves. Let ϕ be an automorphism of the cover, and suppose that $\phi(Q) = Q$ for some $Q \in Y$.
 - a) Show that $Y \times_X Y$ is a smooth curve, and that the first projection $p_1 : Y \times_X Y \rightarrow Y$ is étale. (Hint: #4.)
 - b) Define $s, t : Y \rightarrow Y \times_X Y$ by $s(Q) = (Q, Q)$ and $t(Q) = (Q, \phi(Q))$. Show that s and t are each sections of p_1 , and that each is an isomorphism onto its image, with inverse equal to p_1 .
 - c) Show that the images of s and of t are closed connected subvarieties of $Y \times_X Y$; are of dimension 1; and intersect. Deduce that these images are equal.
 - d) Conclude that ϕ is the identity.
 - e) In the case of $k = \mathbb{C}$, why is the conclusion of (d) expected?