Math 625 Problem Set #7 Due Friday, April 11, 2003

Read Hartshorne, Chapter III, sections 8-10; and Chapter II, section 9.

1. In Hartshorne, Chapter III, do problems 4.7, 5.7, 7.1, 7.3.
Optional: problems 3.1, 4.1, 4.2, 5.6(a).

2. a) Which smooth connected projective curves C have the following property: If P, Q are distinct points of C, $U = C - \{P\}$, $V = C - \{Q\}$, and $f \in \mathcal{O}(U \cap V)$, then there exist $g \in \mathcal{O}(U)$ and $h \in \mathcal{O}(V)$ such that $f = g - h$.

b) What does this say about $H^1(C, \mathcal{O})$?

3. Show explicitly that $d^2 = 0$ in the definition of Čech cohomology, and hence that $B^p \subset Z^p$.

4. Show explicitly that the sum of the residues of a differential form on $\mathbb{P}^1_k$ is 0, for any algebraically closed field k.

5. Let $n, i, m, q_1, \ldots, q_m$ be integers, with $n, m > 0$ and $i \geq 0$. Let $\mathcal{F} = \bigoplus_{j=1}^{m} \mathcal{O}(q_j)$ on $\mathbb{P}^n_k$ for some algebraically closed field k. Find the dimension of $H^i(\mathbb{P}^n, \mathcal{F})$ explicitly in terms of the integers $n, i, q_1, \ldots, q_m$.

6. Show that $\mathrm{Hom}(\mathcal{O}_X, \mathcal{F})$ is naturally isomorphic to $H^0(X, \mathcal{F})$, for any $\mathcal{O}_X$-module $\mathcal{F}$ on a scheme X. What is $\mathcal{H}om(\mathcal{O}_X, \mathcal{F})$?