Math 625 Problem Set #5 Due Thursday, March 25, 2021

Read Hartshorne, Chapter III, Sections 1-4.

1. In Hartshorne, Chapter III, do problems 2.1(a), 3.2, 4.3.
Optional problems from Hartshorne: In Chapter IV, problems 5.2, 6.1. In Chapter III, problems 2.2, 4.4, 4.5.

2. Show explicitly that $d^2 = 0$ in the definition of Čech cohomology, and hence that $B^p \subseteq Z^p$.

3. a) Which smooth connected projective curves C have the following property: If P, Q are distinct points of C, $U = C - \{P\}$, $V = C - \{Q\}$, and $f \in \mathcal{O}(U \cap V)$, then there exist $g \in \mathcal{O}(U)$ and $h \in \mathcal{O}(V)$ such that $f = g - h$.
b) What does this say about $H^1(C, \mathcal{O})$?

4. Show that $\mathrm{Hom}(\mathcal{O}_X, \mathcal{F})$ is naturally isomorphic to $H^0(X, \mathcal{F})$, for any $\mathcal{O}_X$-module $\mathcal{F}$ on a scheme X. What is $\mathcal{H}om(\mathcal{O}_X, \mathcal{F})$? What is $H^0(X, \mathcal{H}om(\mathcal{O}_X, \mathcal{F}))$?

5. Show explicitly that the sum of the residues of a differential form on $\mathbb{P}^1_k$ is 0, for any algebraically closed field k.