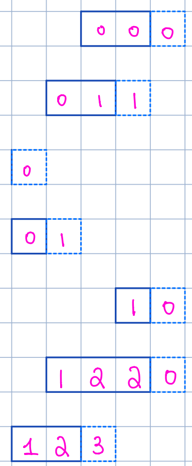
Def: For $\eta, \tau \in \mathbb{N}^m$, define

$$d(\eta,\tau) = \sum_{1\le j<r\le m} \left| [\eta_j, \eta_j+\tau_j] \cap [\eta_r, \eta_r+\tau_r-1] \right|$$

(To count, we can draw rows $[\eta_k, \eta_k+\tau_k]$ $\forall 1\le k\le \ell$. Then fix a box and count the no. of real boxes above it) (including right virtual box)

e.g. $\eta = (0,1,3,0,0,1,2)$, $\tau = (2,3,1,1,0,2,2)$ ← row lengths

$\eta+\tau = (2,4,4,1,0,3,4)$



← The number in each box counts the number of real box above it

$d(\eta,\tau)$ = sum of these numbers = 15.

Def: For any vector η of length n and $I \subseteq [n]$, define

$$h_I(\eta) = \left| \{ (r,s) : 1\le r<s\le n,\ r\in I,\ s\notin I,\ \eta_s = \eta_r + 1 \} \right|$$

e.g. $n=7$, $\eta = (0,1,3,0,0,1,2)$, $\tau = (2,3,1,1,0,2,2)$ $I = \{2,3,6\}$

Hence (r,s) s.t. $1\le r<s\le n$, $r\in I$, $s\notin I$ are:

$(2,4), (2,5), (2,7), (3,4), (3,5), (3,7), (6,7)$

For $\eta = (0, \overset{\text{no choice}}{1, 3}, 0, 0, 1, 2)$ fix •, find non-'•' number on its right which has value greater than it by exactly 1

$\therefore h_I(\eta) = 2$ ← $(2,7)$ and $(6,7)$

For $\tau = (2,3,1,1,0,2,2)$, $h_I(\tau) = 1$

Theorem 5.1.1: For $0\le \ell < m \le N$, we have

$$\underbrace{\langle z^{N-m} \rangle \sum_{\substack{\lambda\in D_N \\ P\in L_{N,\ell}(\lambda)}} t^{|\delta/\lambda|} \prod_{\substack{1<i\le N \\ c_i(\lambda)=c_{i-1}(\lambda)+1}} (1+zt^{-c_i(\lambda)})\, q^{\mathrm{dinv}(P)} x^{\mathrm{wt}_+(P)}}_{\text{c.f. §2}} = \sum_{\substack{J\subseteq[m-1] \\ |J|=\ell}} \sum_{\substack{\tau, (0,\vec{\alpha})\in\mathbb{N}^m \\ |\tau|=N-m}} t^{|\vec{\alpha}|} q^{d((0,\vec{\alpha})+\tau)+h_J(\vec{\alpha})} \underbrace{N_{\beta/\alpha}(X;q)}_{\text{c.f. §5.2}}$$

where $\beta = (0,\vec{\alpha}) + (\underbrace{1,1,\dots,1}_{m}) + \tau$, $\alpha = (\vec{\alpha}, 0) + \varepsilon_J$

$\varepsilon_J = \sum_{j\in J} \varepsilon_j$ = (0,1)-sequence with 1 on the j^{th} pos. iff $j\in J$