Toward Butler's conjecture

joint w/ Donghyun Kim and Seung Jin Lee

- Macdonald polynomials

$\{m_\lambda\}$ $\{e_\lambda\}$ $\{h_\lambda\}$ $\{p_\lambda\}$ $\underset{\text{bases}}{\subseteq}$ Sym

$$\underset{S_\mu}{\text{Schur}} \xrightarrow[q,t \text{ generalization}]{} \underset{P_\mu}{\text{Macdonald } P} \xrightarrow[\text{scaling}]{} \underset{J_\mu}{\text{Macdonald } J}$$

(integral form)

<u>Macdonald positivity conjecture</u> $\mu \vdash n$, let

$$J_\mu[X;q,t] = \sum_{\lambda \vdash n} K_{\lambda\mu}(q,t)\, S_\lambda[(1-t)X]$$

then $K_{\lambda\mu}(q,t) \in \mathbb{N}[q,t]$

q,t - Kostka polynomial

Garsia and Haiman introduced the <u>modified Macdonald polynomial</u> $\tilde{H}_\mu[X;q,t]$

$$\tilde{H}_\mu[X;q,t] = t\text{-rev}\left(J_\mu\left[\frac{X}{1-t};q,t\right]\right)$$

<u>Macdonald positivity conjecture (rephrased)</u> Let $\mu \vdash n$ and

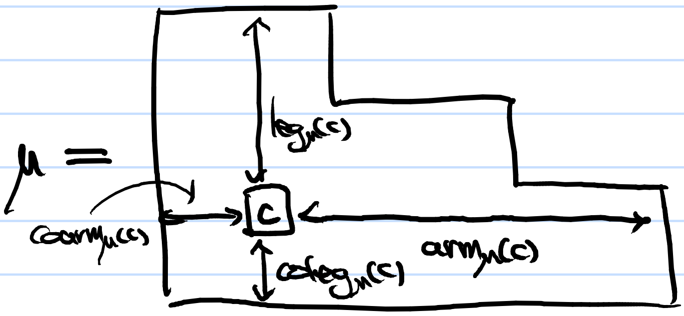
$$\tilde{H}_\mu[X;q,t] = \sum_{\lambda \vdash n} \tilde{K}_{\lambda,\mu}(q,t)\, S_\lambda$$

then $\tilde{K}_{\lambda,\mu}(q,t) \in \mathbb{N}[q,t]$

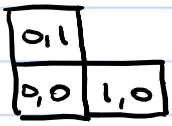
(modified) q,t — Kostka polynomials

- **Garsia–Haiman module V_μ**

$$\Delta_\mu := \det\left(x_i^{\mathrm{coarm}_\mu(c)} \, y_i^{\mathrm{coleg}_\mu(c)} \right)_{\substack{1\le i\le n \\ c\in\mu}} \in \mathbb{C}[x_1,\dots,x_n,y_1,\dots,y_n]$$

$\mu =$ 

eg. $$\Delta_{(2,1)} = \det\begin{bmatrix} 1 & x_1 & y_1 \\ 1 & x_2 & y_2 \\ 1 & x_3 & y_3 \end{bmatrix} = x_2y_3 - x_3y_2 - x_1y_3 + x_3y_1 + x_1y_2 - x_2y_1$$



The <u>Garsia–Haiman module</u> V_μ is defined as

$$V_\mu := \mathrm{Span}\{\text{partial derivatives of } \Delta_\mu\} \subseteq \mathbb{C}[x_1,\dots,x_n,y_1,\dots,y_n]$$

eg. $V_{(2,1)} = \mathrm{Span}\{\ \Delta_{(2,1)}$ $\quad (1,1) \quad S^{(1,1,1)}$

$$\left.\begin{aligned} \partial_{y_1}\Delta_{(2,1)} &= x_3 - x_2 \\ \partial_{y_2}\Delta_{(2,1)} &= x_1 - x_3 \\ \partial_{y_3}\Delta_{(2,1)} &= x_2 - x_1 \end{aligned}\right\} \quad (1,0) \quad S^{(2,1)}$$

$$\left.\begin{aligned} \partial_{x_1}\Delta_{(2,1)} &= y_2 - y_3 \\ \partial_{x_2}\Delta_{(2,1)} &= y_3 - y_1 \\ \partial_{x_3}\Delta_{(2,1)} &= y_1 - y_2 \end{aligned}\right\} \quad (0,1) \quad S^{(2,1)}$$

$$\partial_{x_1}\partial_{y_2}\Delta_{(2,1)} = 1 \quad (0,0) \ \} \quad S^{(3)}$$

$$\dim V_{(2,1)} = 6 \overset{!}{=} 3!$$

Note that V_μ is a <u>bigraded</u> <u>S_n-module</u>.

(x-, y- degree) $\quad \begin{pmatrix} \sigma x_i = x_{\sigma(i)} \\ \sigma y_i = y_{\sigma(i)} \end{pmatrix}$

$$\mathrm{Hilb}(V_{(2,1)}; q,t) = qt + 2q + 2t + 1$$

(**Recall** $\mathrm{Frob}: \{S_n\text{-modules}\} \longrightarrow \mathrm{Sym}$

$S^\lambda \longmapsto s_\lambda$

irreducible module $\quad$ Schur function)

$$\mathrm{grFrob}(V_{(2,1)}; q,t) = qt\, s_{111} + (q+t)\, s_{21} + s_3$$
$$\overset{!}{=} \tilde{H}_{(2,1)}[X; q,t]$$

Thm (Haiman 01, $n!$-theorem, conj. by Garsia-Haiman 93/96)

For $\mu \vdash n$, we have

① $\dim(V_\mu) = n!$

② $\mathrm{grFrob}(V_\mu; q,t) = \tilde{H}_\mu[X; q,t]$

$\left(\because\ \mathrm{grFrob}(V_\mu; 1,1) = h_{(1^n)}\right)$

In particular, $\tilde{H}_\mu$ is Schur positive.

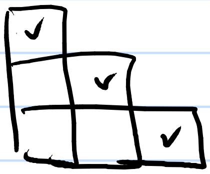
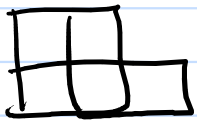
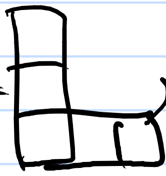
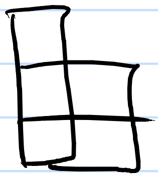
Open problem Find a combinatorial interpretation of $\tilde{K}_{\lambda\mu}(q,t)$

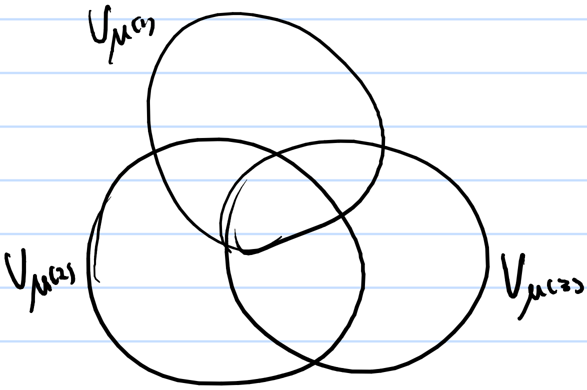
Rmk $\tilde{K}_{\lambda,\mu}(1,1) = \#\,\mathrm{SYT}(\lambda)$

$$\tilde{K}_{\lambda,\mu}(q,t) = \sum_{T \in \mathrm{SYT}(\lambda)} q^{\mathrm{stat}_q(T)}\, t^{\mathrm{stat}_t(T)}$$

- Science Fiction conjecture.

Let $\mu \vdash n+1$, $\mu^{(1)}, \cdots, \mu^{(k)} \subseteq \mu$ k distinct partitions such that $|\mu/\mu^{(1)}| = \cdots = |\mu/\mu^{(k)}| = 1$

e.g. $\mu =$  $\mu^{(1)} =$  $\mu^{(2)} =$  $\mu^{(3)} =$ 



Conj (Bergeron – Garsia 99)

- ($\frac{n!}{k}$ – conjecture)

$$\dim\left(\bigcap_{i=1}^{k} V_{\mu^{(i)}}\right) = \frac{n!}{k}$$

- $$\operatorname{grFrob}\left(\bigcap_{i=1}^{k} V_{\mu^{(i)}} ; q,t\right) = \sum_{i=1}^{k} \prod_{j\neq i} \frac{T_{\mu^{(j)}}}{T_{\mu^{(j)}} - T_{\mu^{(i)}}} \tilde{H}_{\mu^{(i)}}$$

$$=: I_{\mu^{(1)},\cdots,\mu^{(k)}}[X; q,t]$$ (Macdonald intersection polynomial)

where $T_\mu := \prod_{c\in\mu} q^{\mathrm{coarm}_\mu(c)} t^{\mathrm{coleg}_\mu(c)}$

- $V_{(2,1)} = \mathrm{Span}\{\Delta_{(2,1)},\ x_1-x_2,\ x_2-x_3,\ y_1-y_2,\ y_2-y_3,\ 1\}$

$$\tilde{H}_{(2,1)} = qt\, s_{111} + (q+t)\, s_{21} + 1\, s_3$$

$$U_{(3)} = \mathrm{Span}\{\overset{(3,0)}{\Delta_{(3)}},\ \overset{(2,0)}{\underline{(2x_1-x_2-x_3)(x_2-x_3),\ (2x_2-x_3-x_1)(x_3-x_2)}},\ \underbrace{x_1-x_2,\ x_2-x_3}_{(1,0)},\ \underbrace{1}_{(0,0)}\}$$

$$\tilde{H}_{(3)} = q^3 s_{111} + (q^2+q)\, s_{21} + 1\, s_3$$

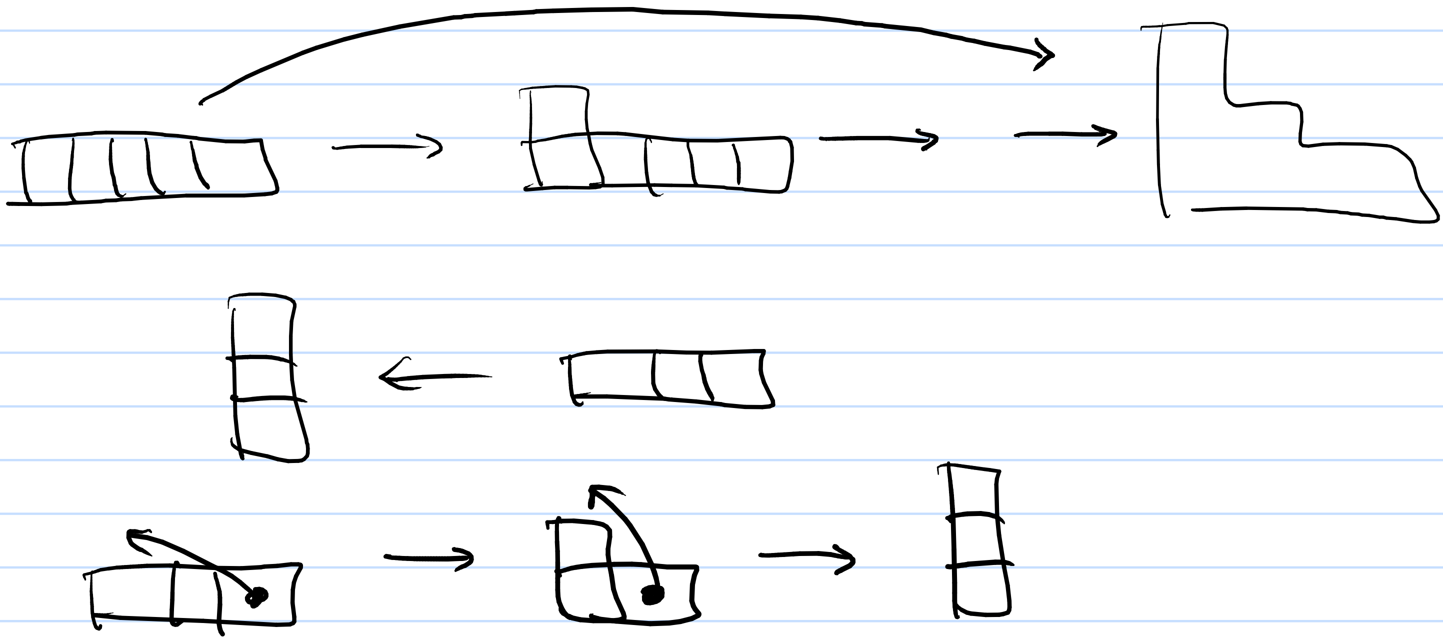
$$\dim(V_{(2,1)} \cap V_{(3)}) = 3 = \frac{3!}{2}$$

$$\mathrm{grFrob}\left(V_{(2,1)} \cap V_{(3)}\right) = q\, s_{21} + 1\, s_3$$

On the other hand

$$I_{(2,1),(3)} = \frac{T_{(3)}\tilde{H}_{(2,1)} - T_{(2,1)}\tilde{H}_{(3)}}{T_{(3)} - T_{(2,1)}} = q\, s_{21} + s_3$$

$$T_{(3)} = \boxed{1}\boxed{q}\boxed{q^2} = q^3, \quad T_{(2,1)} = \begin{array}{|c|c}\hline t & \\ \hline 1 & q \\ \hline \end{array} = qt$$



- HHL formula

Thm (Haglund–Haiman–Loehr 04) For $\mu \vdash n$,

$$\tilde{H}_\mu[X;q,t] = \sum_{\sigma \in S_n} q^{\mathrm{inv}_\mu(\sigma)} t^{\mathrm{maj}_\mu(\sigma)} F_{\mathrm{iDes}(\sigma)}$$

Thm (Kim–Lee–O, 22+) For $\nu \vdash n+1$, $\lambda, \mu \subseteq \nu$ s.t. $|\nu/\lambda| = |\nu/\mu| = 1$. Then

$$I_{\lambda,\mu} = \sum_{\sigma \in B_{\lambda,\mu}} q^{\mathrm{inv}_\mu(\sigma)} t^{\mathrm{maj}_\mu(\sigma)} F_{\mathrm{iDes}(\sigma)}$$

Butler permutations, and this is of size $\frac{n!}{2}$

Conj (Butler 94)

$$I_{\lambda,\mu}[X;q,t] = \frac{T_\lambda \tilde{H}_\mu - T_\mu \tilde{H}_\lambda}{T_\lambda - T_\mu} \text{ is Schur positive.}$$

Rmk SF $\Longrightarrow$ Butler's conjecture

Thm (Kim-Lee-O. 22+)

- $I_{\lambda,\mu}[X;q,t]$ is Schur positive if

  $\nu \backslash \lambda$ is in the first row / second row of ν.

- $I_{\lambda,\mu}[X;q,1]$ is Schur positive

  ($I_{\lambda,\mu}[X;1,t]$ is Schur positive)

Thm (Kim-Lee-O. 22+)

$$I_{\lambda,\mu}[X;1,1] = h_{(2,1^{n-2})}$$

- Combinatorial formula for $\widetilde{K}_{\lambda,\mu}(q,t)$ which is compatible with Butler's conjecture for μ= hook / μ= 2-column

$$\xrightarrow[\text{Vetter}]{} \quad \mu = (m, 2^a, 1^b)$$

- Diagonal coinvariants

The diagonal coinvariant algebra

$$DR_n := \mathbb{C}[x_1 \dots x_n, y_1 \dots y_n] \Big/ \langle \mathbb{C}[x_1 \dots x_n, y_1 \dots y_n]^{S_n}_{+} \rangle$$

Thm (Haiman 02, $(n+1)^{n-1}$ - theorem)

$$\begin{cases} \dim(DR_n) = (n+1)^{n-1} \\ \mathrm{grFrob}(DR_n; q,t) = \nabla e_n \end{cases}$$

$\nabla \tilde{H}_\mu = T_\mu \tilde{H}_\mu$ (for ∇); e_n: nth elementary symm. fcn.

Thm (Carlsson-Mellit 18, Shuffle theorem conj. by Haglund-Haiman-Loehr-Remmel-Ulyanov 05)

$$\nabla e_n = D_n[X; q,t]$$

$$:= \sum_{(\pi,w) \in \mathrm{Park}_n} q^{\mathrm{dinv}(\pi,w)} t^{\mathrm{bounce}(\pi)} F_{i\mathrm{Des}(w)}$$

- Rational shuffle — Mel 18
- Delta theorem — Mel-D'Add 22, HMBPS 23
- Loehr-Warrington — HMBPS 23+

Thm (Kim–Lee–O. 23+) $\mu \vdash n+1$, $\mu^{(1)}, \dots, \mu^{(k)} \subseteq \mu$

s.t. $|\mu/\mu^{(1)}| = \cdots = |\mu/\mu^{(k)}| = 1.$

(a) (Vanishing identity) For $m < k-1$, then we have

$$e_{n-m}^{\perp} I_{\mu^{(1)}, \dots, \mu^{(k)}} = 0$$

(b) (Connection to ∇e_n)

$$e_{n-k+1}^{\perp} I_{\mu^{(1)}, \dots, \mu^{(k)}} = T_{\bigcap_{i=1}^{k} \mu^{(i)}} \nabla e_{k-1}$$

(c) (Connection to $D_n[X;q,t]$)

$$e_{n-k+1}^{\perp} I_{\mu^{(1)}, \dots, \mu^{(k)}} = T_{\bigcap_{i=1}^{k} \mu^{(i)}} D_{k-1}[X;q,t]$$

Cor Shuffle theorem.

Thm (Kim–Lee–Oh 23+)

$$I_{\mu^{(1)} \dots \mu^{(k)}}[X;1,1] = \sum_{\lambda \vdash k-1} (-1)^{k-1-\ell(\lambda)} \mathrm{Krew}(\lambda + (1^{\ell(\lambda)})) \, h_{\lambda + (1^{n-k+1})}$$

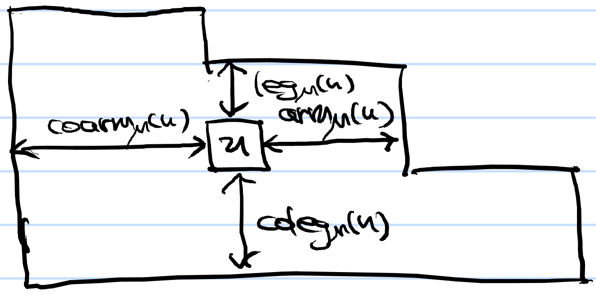
- Review HHL formula
- filled diagram and its Macdonald polynomials
- column exchange rule

For $S \subseteq [n-1]$, the <u>fundamental quasisymmetric function</u>

$$F_{n,S} = F_S := \sum_{\substack{i_1 \le \cdots \le i_n \\ j \in S \Rightarrow i_j < i_{j+1}}} x_{i_1} \cdots x_{i_n}$$

- Haglund–Haiman–Loehr formula

$\mu =$



For a partition μ, a pair (u,v) of cells in μ is <u>attacking pair</u> if either

- $\boxed{u} \ \cdots \ \boxed{v}$ or
- $\begin{array}{ccc} & \cdots & \boxed{u} \\ \boxed{v} & \cdots & \end{array}$

For a filling σ of μ, a pair (u,v) forms an <u>inversion pair</u> if

- (u,v): attacking pair $\quad \sigma(u) > \sigma(v)$.

Denote the set of inversion pairs of σ by $\mathrm{Inv}_\mu(\sigma)$ and

$$\mathrm{inv}_\mu(\sigma) := \prod_{(u,v) \in \mathrm{Inv}_\mu(\sigma)} q$$

eg. $\mathrm{inv}_\mu\left(\begin{array}{|c|c|c|} \hline 3 & & \\ \hline 2 & 5 & 7 \\ \hline 6 & 1 & 4 \\ \hline \end{array}\right) = \mathrm{inv}_\mu(3257614) = q^3$

A cell u is a <u>descent</u> of σ if $\sigma(u) > \sigma(v)$, where $v =$ cell right below u.

Denote the set of descents of σ by $\mathrm{Des}_\mu(\sigma)$ and

$$\mathrm{maj}_\mu(\sigma) := \prod_{u \in \mathrm{Des}_\mu(\sigma)} q^{-\mathrm{arm}_\mu(u)}\, t^{\mathrm{leg}_\mu(u)+1}$$

eg.
$$\mathrm{maj}_\mu\left(\begin{array}{|c|c|c|} \hline 3 & & \\ \hline 2 & 5 & 7 \\ \hline 6 & 1 & 4 \\ \hline \end{array}\right) = \mathrm{maj}_\mu(3257614) = q^{-1}t^3$$

with the descents t (above 3), $q^{-1}t$ (above 5), t (above 7).

$$\mathrm{stat}_\mu(\sigma) := \mathrm{inv}_\mu(\sigma)\cdot \mathrm{maj}_\mu(\sigma) \underset{\text{Example}}{=} q^2t^3$$

<u>Thm</u> (Haglund–Haiman–Loehr 05) For $\mu \vdash n$,

$$\tilde{H}_\mu[X;q,t] = \sum_{\sigma \in S_n} \mathrm{stat}_\mu(\sigma)\, F_{i\mathrm{Des}(\sigma)}$$

eg. $\mu = (2,1)$

σ	$\begin{smallmatrix}1\\2\,3\end{smallmatrix}$	$\begin{smallmatrix}2\\1\,3\end{smallmatrix}$	$\begin{smallmatrix}2\\3\,1\end{smallmatrix}$	$\begin{smallmatrix}1\\3\,2\end{smallmatrix}$	$\begin{smallmatrix}3\\1\,2\end{smallmatrix}$	$\begin{smallmatrix}3\\2\,1\end{smallmatrix}$
$\mathrm{inv}_\mu(\sigma)$	1	1	q	q	1	q
$\mathrm{maj}_\mu(\sigma)$	1	t	1	1	t	t
$\mathrm{stat}_\mu(\sigma)$	1	t	q	q	t	qt
$i\mathrm{Des}(\sigma)$	ϕ	$\{1\}$		$\{2\}$		$\{1,2\}$

$$\tilde{H}_\mu = F_\phi + (q+t)F_{\{1\}} + (q+t)F_{\{2\}} + qtF_{\{1,2\}}$$
$$= s_3 + (q+t)s_{21} + qt\, s_{111}$$

<u>Recall</u> Our goal is to compute

$$I_{\lambda,\mu} = \frac{T_\lambda \tilde{H}_\mu - T_\mu \tilde{H}_\lambda}{T_\lambda - T_\mu}$$

We may hope that for $\sigma \in S_n$, we have either

① $\mathrm{stat}_\mu(\sigma) = \mathrm{stat}_\lambda(\sigma)$ or

② $T_\lambda \, \mathrm{stat}_\mu(\sigma) = T_\mu \, \mathrm{stat}_\lambda(\sigma)$

Then $$I_{\lambda,\mu} = \sum_{\substack{\sigma \in S_n \\ \text{satisfying ①}}} \mathrm{stat}_\lambda(\sigma) F_{\mathrm{iDes}}$$

But it's not the case.

eg. $\mu = (2,1)$ $\lambda = (3) = \boxed{}\boxed{}\boxed{}$

σ	123	213	231	132	312	321
$\mathrm{stat}_\mu(\sigma)$	1	t	① q	q	t	qt
	1①			1①	1②	1②
$\mathrm{stat}_\lambda(\sigma)$	1	q	② q^2	q	q^2	q^3
$\mathrm{iDes}(\sigma)$	ϕ	$\{1\}$		$\{2\}$		$\{1,2\}$

Can we modify the combinatorial formula for $\tilde{H}_\mu, \tilde{H}_\lambda$ to "swap" the stat properly

A <u>diagram</u> (D) is a collection of cells.

A <u>filled diagram</u> is a pair (D, f) of a diagram D and a filling function f

$$f : D \setminus \{\text{bottom cells of } D\} \longrightarrow \mathbb{F}$$

where $\mathbb{F}$ is a field (usually contains $\mathbb{Q}(q,t)$)

For a filling σ of D, $\mathrm{inv}_D(\sigma)$ and $\mathrm{Des}_\sigma(D)$ similarly to $\mathrm{inv}_\mu(\sigma)$ and $\mathrm{Des}_\mu(\sigma)$

$$\mathrm{maj}_{(D,f)}(\sigma) := \prod_{u \in \mathrm{Des}_D(\sigma)} f(u)$$

$$\mathrm{stat}_{(D,f)}(\sigma) := \mathrm{inv}_D(\sigma) \cdot \mathrm{maj}_{(D,f)}(\sigma)$$

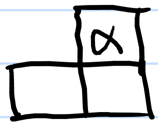
Finally, we define the (generalization) modified Macdonald polynomial $\tilde{H}_{(D,f)}$ is defined as

$$\tilde{H}_{(D,f)} := \sum_{\sigma \in S_n} \mathrm{stat}_{(D,f)}(\sigma)\, F_{\mathrm{iDes}(\sigma)}$$

If we let $D = \mu$ and $f^{st}_\mu(u) = q^{-\mathrm{arm}_\mu(u)}\, t^{\mathrm{leg}_\mu(u)+1}$

then we have

$$\tilde{H}_{(D,f)} = \tilde{H}_\mu$$

eg. $(D,f) =$ 

σ	$\begin{smallmatrix}1\\23\end{smallmatrix}$	$\begin{smallmatrix}2\\13\end{smallmatrix}$	$\begin{smallmatrix}2\\31\end{smallmatrix}$	$\begin{smallmatrix}1\\32\end{smallmatrix}$	$\begin{smallmatrix}3\\12\end{smallmatrix}$	$\begin{smallmatrix}3\\21\end{smallmatrix}$
$\mathrm{inv}_D(\sigma)$	1	q	q	q	q	q^2
$\mathrm{maj}_{(D,f)}(\sigma)$	1	1	α	1	α	α
$\mathrm{stat}_{(D,f)}(\sigma)$	1	q	$q\alpha$	q	$q\alpha$	$q^2\alpha$
$\mathrm{iDes}(\sigma)$	ϕ	$\{1\}$		$\{2\}$		$\{1,2\}$

$$\tilde{H}_{(D,f)} = F_{\phi} + (q+q\alpha)F_{\{1\}} + (q+q\alpha)F_{\{2\}} + q^2\alpha F_{\{1,2\}}$$

Rmk weighted characteristic polynomial (CM(8)) $= \tilde{H}_{(D,f)}$

$$\chi_{(\pi,wt)} := \sum_{\sigma\in S_n} \prod_{\substack{(i,j)<\pi \\ \sigma(i)>\sigma(j)}} q \prod_{\substack{c=(i,j)\in \mathrm{Corner}(\pi) \\ \sigma(i)>\sigma(j)}} wt(c)\, F_{\mathrm{iDes}(\sigma)}$$

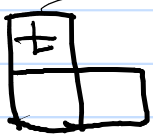
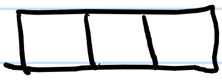
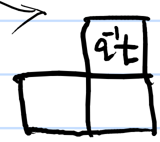
$(D,f) \xrightarrow[\text{associate}]{} (\pi, wt)$ so that we have

$$\tilde{H}_{(D,f)} = \chi_{(\pi,wt)}$$

It is possible that for two distinct filled diagrams (D,f) and (D',f') and

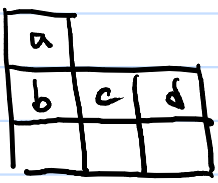
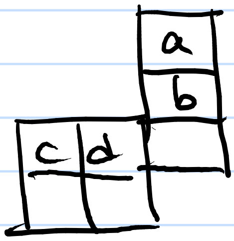
$$\tilde{H}_{(D,f)} = \tilde{H}_{(D',f')}$$

eg $\mu=(2,1)$ $\lambda=(3)$

$(\mu, f_\mu^{st}) =$  $(\lambda, f_\lambda^{st}) =$  $(D,f) =$ 

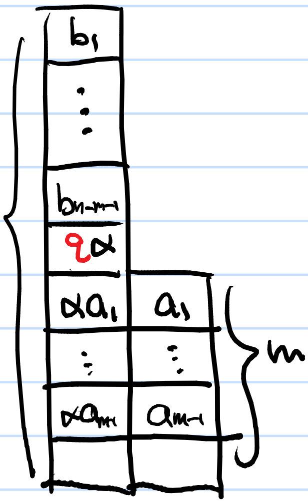
σ	123	213	231	132	312	321
$\mathrm{stat}_{(\mu, f_\mu^{st})}(\sigma)$	1	t	q	q	t	qt
$\mathrm{stat}_{(\lambda, f_\lambda^{st})}(\sigma)$	1	q	q^2	q	q^2	q^3
	1 ①	1 ①	1 ②	1 ①	1 ②	1 ②
$\mathrm{stat}_{(D,f)}(\sigma)$	1	q	t	q	t	qt

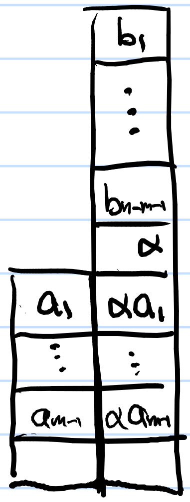
Lemma (cycling Lemma)

 = 

- Column exchange rule

Prop

$(\mu, f_\mu) =$ 

$(\lambda, f_\lambda) =$ 

Then there is a (stat, iDes, content)-preserving bijection

$\phi_{n,m} : S_{n+m} \longrightarrow S_{n+m}$, i.e. $\phi_{n,m}$ satisfies

(ϕ1) $\mathrm{stat}_{(\mu,f_\mu)}(\sigma) = \mathrm{stat}_{(\lambda,f_\lambda)}(\phi_{n,m}(\sigma))$

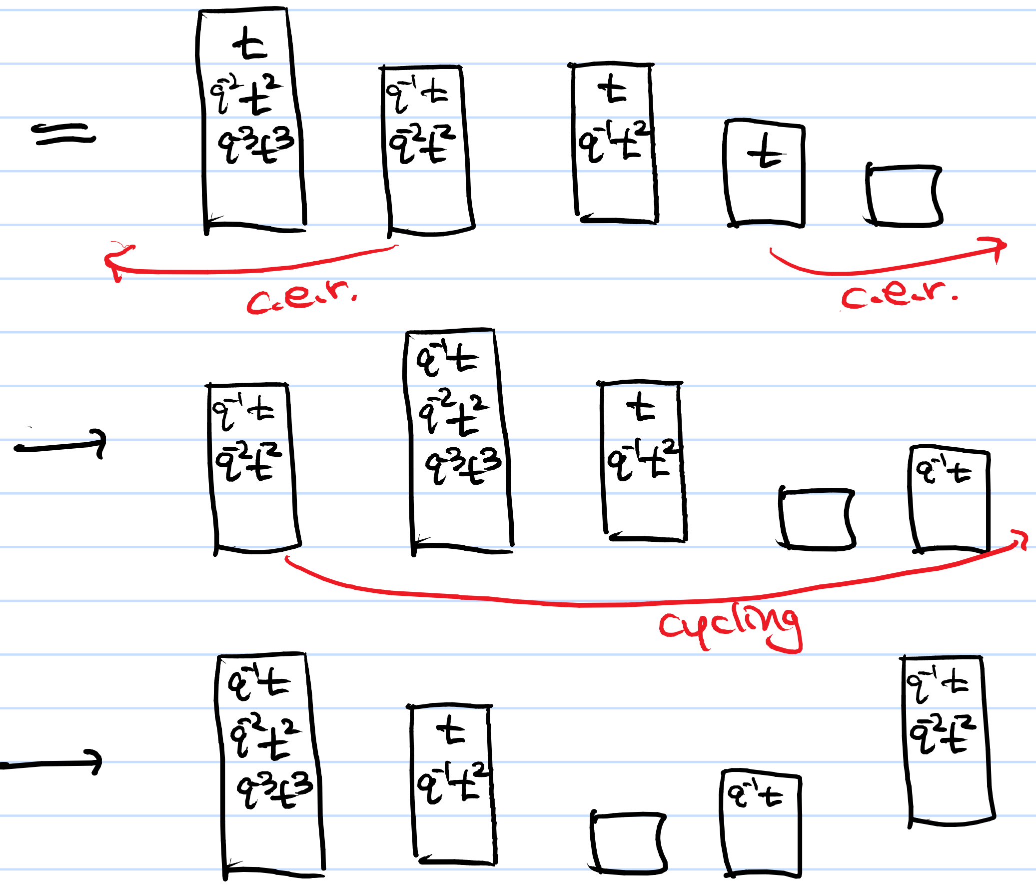
(ϕ2) $\mathrm{iDes}(\sigma) = \mathrm{iDes}(\phi_{n,m}(\sigma))$

(ϕ3) $\begin{cases} \sigma(i) = (\phi_{n,m}(\sigma))(i) \quad \text{for } 1 \le i \le n-m \\ \{\sigma(n-m+2i-1), \sigma(n-m+2i)\} = \{(\phi_{n,m}(\sigma))(n-m+2i-1), \\ \qquad (\phi_{n,m}(\sigma))(n-m+2i)\} \quad \text{for } 1 \le i \le m \end{cases}$

Rmk (ϕ1) and (ϕ2) $\Longrightarrow$ $\widetilde{H}_{(\mu,f_\mu)} = \widetilde{H}_{(\lambda,f_\lambda)}$

eg. $\mu = (5,4,3,1)$ $\quad f_\mu = f_\mu^{st}$

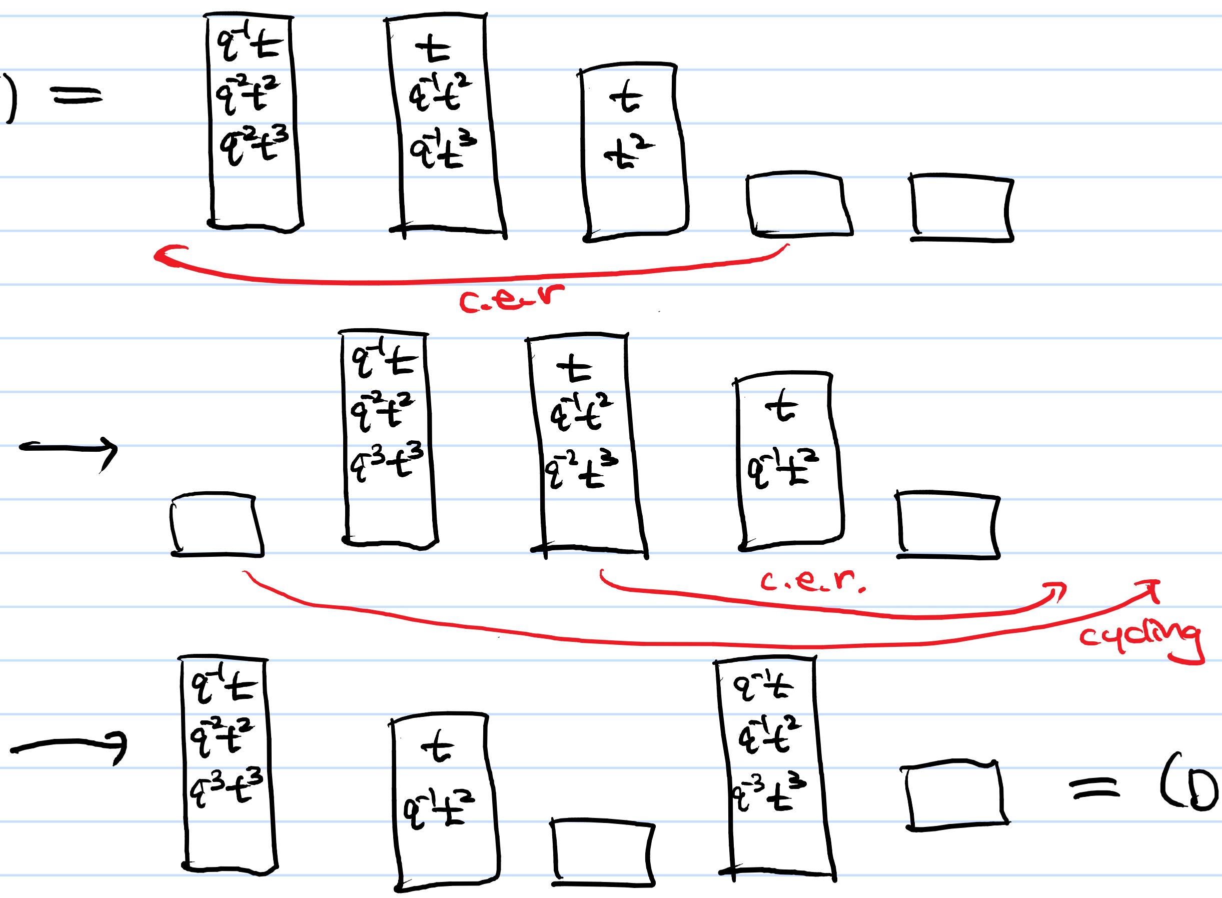
$(\mu, f_\mu) =$



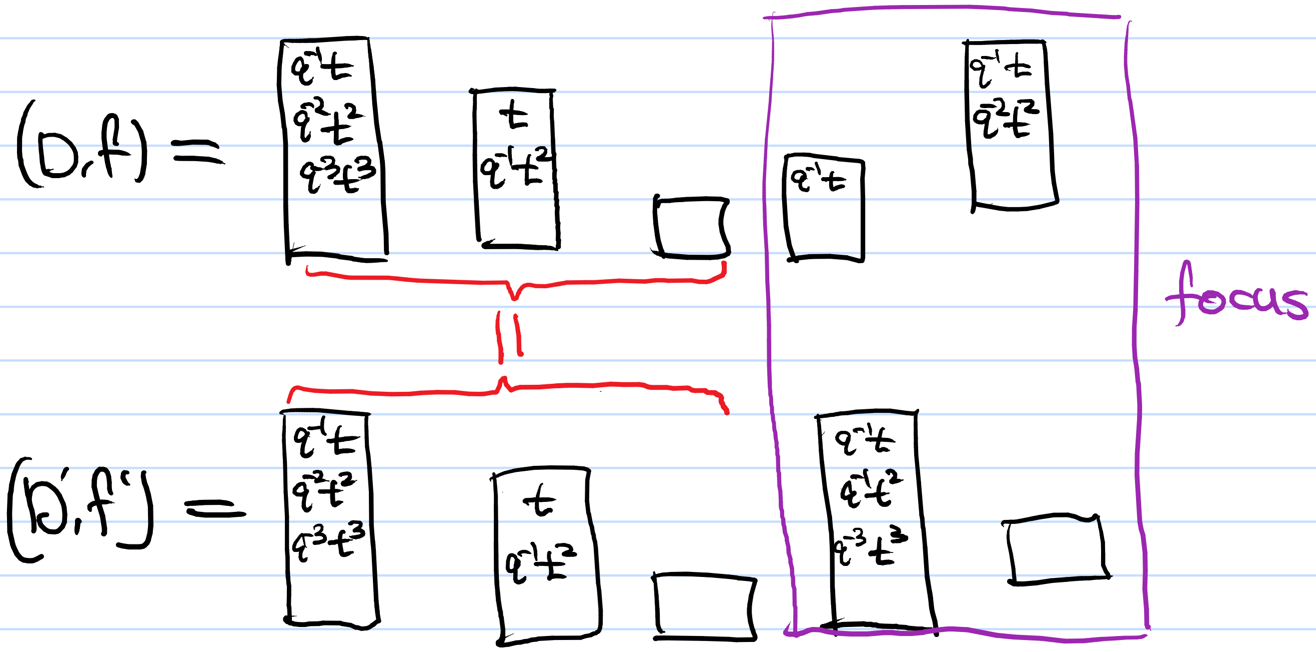
$= (D, f)$

$\lambda = (5,3,3,2)$ $\quad f_\lambda = f_\lambda^{st}$

$(\lambda, f_\lambda^{st}) =$



$= (D', f')$



Rmk The identity $\tilde{H}_{(\mu, f_\mu^{st})} = \tilde{H}_{(D,f)}$ also comes from Thm 5.1.1 of Haglund – Haiman – Loehr 08 where they studied a combinatorial formula for nonsymmetric Macdonald polynomials.

- Our proof is bijective answering a question of Haglund

- (Φ1) and (Φ2) is enough to prove the identity $\Phi_{n,m}$ also satisfies (Φ3) which makes our theorem stronger version of HHL08 and this fact is crucial in the later proof

- The setting is more flexible.