Midterm Exam - Math 170

June 18, 2015

1. Why must a triangle always have at least one of its angles $60°$ or more?

2. Please factor the following numbers into products of primes: 288, 693, 2048

3. Does a non-prime divided by a non-prime ever result in a prime? Always? Sometimes? Never? Please explain and give examples to support your answers.

4. Using the reasoning from the proof that there are infinitely many prime numbers, show that the list of primes 11, 13 is incomplete. Which new primes do we find as a result?

5. What is 50^{50} *mod* 7?

6. The exam today began at 4:30 PM. What time will it be 495 hours after the exam began? (Hint: What time will it be 4815 hours or 48015 or 480000015 hours after the exam began, if we disregard clock changes for daylight savings time etc.?)

7. Explain why if $a \equiv b \ mod \ n$ and $b \equiv c \ mod \ n$ then $a \equiv c \ mod \ n$.

8. In this problem, you will prove that $\sqrt{5}$ is an irrational number by completing the steps below. Like when we proved that $\sqrt{2}$ is irrational, this will be a *proof by contradiction*. We begin by assuming that $\sqrt{5}$ is rational: $\sqrt{5} = \frac{a}{b}$, with a and b natural numbers. By the end of the proof you will have shown that this initial assumption leads to a contradiction, so it must be false.

   a) True or False: If $\sqrt{5}$ is a rational number, then we can assume it is in lowest terms, so that the only common divisor of a and b is 1.

   b) Show that if $\sqrt{5} = \frac{a}{b}$, then $a^2 = 5b^2$.

   c) Explain why, if 5 divides a^2, 5 must divide a. (Hint: What is the relationship between the prime factorization of a and that of a^2?)

   d) Since 5 divides a, show that 25 divides a^2.

   e) From part d), deduce that 5 divides b^2 and that therefore, as in part c), 5 must divide b.

   f) Explain why this is a contradiction, and hence why $\sqrt{5}$ cannot be a rational number.

9. Write a paragraph about one of the two applications of modular arithmetic that we have studied: Error detection for UPCs, ISBNs, bank codes etc. or RSA public key encryption. Explain in your own words how it works and something you found interesting about it.