Math 202 Jerry L. Kazdan

Homework Set 6 (Due in class on Thursday, Oct. 29)

(late papers accepted until 1:00 Friday)

The problem numbers refer to the D'Angelo-West text.

1. [#14.24] Let $f(x) := x^2 - 4x + 6$ and let x_n be a sequence defined by the recurrance $x_{n+1} = f(x_n)$, that is, $x_{n+1} = x_n^2 - 4x_n + 6$.
   a) If $\lim_{n\to\infty}$ exists and equals L, what possible values can L have?
   b) The behavior as $n \to \infty$ depends on the initial value, x_0. For each $x_0 \in \mathbb{R}$, describe this behavior. [HINT: Graph the functions $y = x$ and $y = f(x)$ and interpret the graphs. In this example, it may help to rewrite $y = f(x)$ as $y - 2 = (x-2)^2$.]

2. [#14.27] For $c > 0$, let $x_n = (c^n + 1)^{1/n}$. Determine $\lim x_n$. More generally find $\lim_{n\to\infty}(a^n + b^n)^{1/n}$. [HINT: First consider the case $c < 1$ and use the Squeese Theorem.]

3. [#14.30] Let x_n be the sequence defined recursively by $x_1 = 1$ and $x_{n+1} = 1/(x_1 + \cdots + x_n)$. Prove that this sequence converges and obtain the limit.

4. [#14.33] If $\displaystyle\sum_{k=1}^{\infty} a_k$ converges to A and $\displaystyle\sum_{k=1}^{\infty} b_k$ converges to B, show that $\displaystyle\sum_{k=1}^{\infty}(a_k + b_k)$ converges to $A + B$.

5. [#14.36] Find the rational number whose repeating decimal expansion is $.247247247\ldots$. Also, find the rational number whose repeating octal (that is, base 8) expansion is $.247247247\ldots$.

6. [#14.43] Compute $\sum_{k=1}^{\infty}\left(\frac{x}{x+1}\right)^k$. What assumptions must be made about x?

7. [Telescoping Series #14.44]
   a) Compute $\sum_{n=1}^{N}\frac{1}{n(n+1)}$ and then $\sum_{n=1}^{\infty}\frac{1}{n(n+1)}$.
      HINT: Use the *partial fraction decomposition* $\frac{1}{(x-a)(x-b)} = \frac{1}{a-b}\left[\frac{1}{x-a} - \frac{1}{x-b}\right]$.
   b) Use this to estimate $\sum_{=n1}^{\infty}\frac{1}{n^2}$

8. [#14.45] Let $S_n := a_1 + \cdots + a_n$. If $S_n = 1/n$ for all $n \ge 1$, find a_n for all $n \ge 1$.

[Last revised: October 27, 2009]