[Who Solved the Secretary Problem?]: Comment

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meet condition (b). Moreover, the upper bound for $n = 3, j + 1$ is $(2 + k)/(4 + k)$ which is bigger than ½, so condition (a) cannot be satisfied either.

Here is one more result for Pareto priors: If we reflect the one-sided priors (i.e., look at $-X_1, \cdots, -X_n$), then the lower bound calculations are virtually the same as in the two-sided case, and the result is a slight improvement, to $(n - j)!/(\alpha + n)(\alpha + n - 1) \cdots (\alpha + j + 1)$ for any $\alpha > 0$. For $n = 3, j = 2$, this is an improvement from $1/(k + 4)$ to $1/(\alpha + 3)$—still not good enough to meet condition (b).

Thus the *Ferguson Secretary Problem* remains unsolved. Indeed, from these Pareto prior examples, it is not at all clear what the solution is: do the required exchangeable sequences exist or don't they? This quest for sufficiently "non-informative priors" should interest some Bayesians, too.

ADDITIONAL REFERENCE

HILL, B. M. (1968). Posterior distribution of percentiles: Bayes' theorem for sampling from a population. *J. Amer. Statist. Assoc.* **63** 677–691.

Comment

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I am confused by Tom's attempt to clear up the confusion among various versions of the secretary problem. In Section 2 he defines the simplest form of the problem, in Section 4 he distinguishes secretary problems from Cayley's problem, etc. in which one observes numerical values of some possibly continuous random variable rather than just relative ranks, and in Section 5 he defines the 'general' secretary problem to be "a sequential observation and selection problem in which the payoff depends on the observations only through their relative ranks and not otherwise on their actual values." So far, so good. Then in Section 6 he introduces into the discussion the two-person googol game, which is not a secretary problem, and in Section 7 and Section 8 says that nobody has solved "the" secretary problem, possibly because no one realized that there was a game-theoretical problem to be solved. I can't agree with that.

Consider two cases of the secretary problem: (I) the payoff is 1 if we choose the best of the the n applicants, 0 otherwise, and we want to maximize the expected payoff, and (II) the loss is the absolute rank of the person selected (1 for the best, $\cdots$, n for the worst), and we want to minimize the expected loss. When all $n!$ orders of the applicants are equally likely the solutions of (I) and (II) have been known and published for some time. And when the probabilities of the various permutations are controlled by an antagonist, so that (I) and (II) become game-theoretical (minimax) problems, their solutions are *also* in the literature: See problem 7 on page 60 of Chow, Robbins and Siegmund (1971), and page 89 of Chow, Moriguti, Robbins and Samuels (1964).

In the latter reference it is also shown that when the $n!$ permutations *are* equally likely, the minimal expected loss for (II) with n applicants tends as $n \to \infty$ to the finite limit

$$A_1 = \prod_{j=1}^{\infty} \left(1 + \frac{2}{j}\right)^{1/(1+j)} \cong 3.8695.$$

This surprising result can be obtained by a heuristic argument involving a sequence of differential equations, but the argument is hard to make rigorous. The same heuristic argument yields a more general result: if the loss is taken to be $x(x + 1) \cdots (x + k - 1)$, where x is the absolute rank of the person selected and k is a fixed positive integer, then the minimal expected loss as $n \to \infty$ tends to

$$A_k = k!\left\{\prod_{j=1}^{\infty} \left(1 + \frac{k+1}{j}\right)^{1/(k+j)}\right\}^k.$$

(As $k \to \infty$ the quantity in braces tends to $e^{\pi^2/6} \cong 5.1807$.) But when the loss is x^2, rather than x or $x(x + 1)$, the limit as $n \to \infty$ of the minimal expected loss has not been exhibited explicitly by any formula such as this (it is, of course, less than A_2), nor has the minimax game-theoretical probability distribution of permutations been obtained for this case. Down with googol and up with problems like these!

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ADDITIONAL REFERENCE

CHOW, Y. S., ROBBINS, H. and SIEGMUND, D. (1971). *Great Expectations: The Theory of Optimal Stopping.* Houghton Mifflin, Boston.