Math 210 | Exam 2 | Jerry L. Kazdan
November 20, 2001 | 12:00 — 1:20

DIRECTIONS: This exam has 7 problems (15 points each). To receive full credit your solution must be clear and correct. No fuzzy reasoning. You have 1 hour 20 minutes. Closed book, no calculators, but you may use one 3 $\times$5 with notes on both sides. Please box your answers.
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1. A large number, $N$, of people are subjected to a blood test, the result of which is either "positive" or "negative". It can be given in two ways:
i). Each person can be tested separately, so $N$ tests are required.
ii). The blood samples of $k$ persons can be pooled and analyzed together. If this test is negative, then one test suffices for the $k$ people, while if the test is positive, each of the $k$ people must be tested separately so $k+1$ tests are required for the $k$ people.
Assume that the probability, p, that a test is positive is the same for all people and that these events are all independent.
a). Find the probability that the test for a pooled sample of $k$ people will be positive.
b). What is the expected value of the number of tests necessary under plan ii)? [Assume that $N$ is divisible by $k$].

2. Write a Perl (or Maple) script that uses the "Monte Carlo" method for estimating the area of the ellipse

$$\frac{x^2}{9} + \frac{y^2}{16} \le 1.$$

This ellipse lies inside the box $Q = \{(x,y) : |x| \le 3, |y| \le 4\}$. The Monte Carlo method says to pick many points at random in the box $Q$. The area of the ellipse will correspond to the percentage of the points that in the ellipse.
REMARK: In case it helps, here is an *unrelated* perl script.

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```
#!/usr/bin/perl -w
#-------------------------- What This Does ----------------------------
# If you toss a coin at random N times, how many heads do you get?
#---------------------------- Main Program ----------------------------
$N = 10000;     # number of tosses
$heads = 0;     # initialize

for ($k=0; $k<$N; $k++) {
    $toss = int(rand(2));
    if ($toss == 0) {$heads = $heads +1;}
    if (($k % 1000) == 0) {print "In $k tosses there were $heads heads.\n";}
}
print "\n   Summary: In $N tosses there were $heads heads.\n";
```

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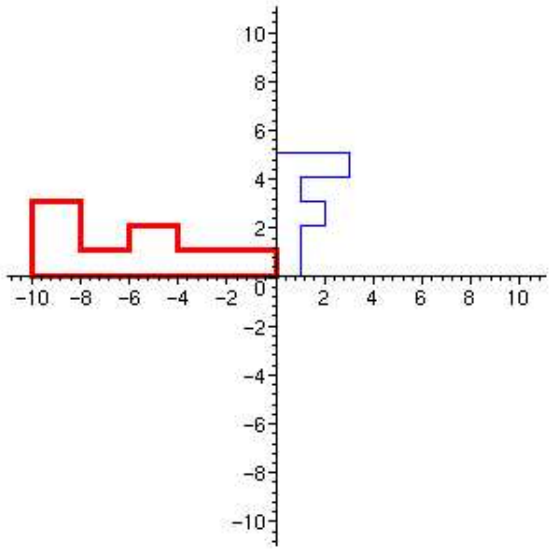
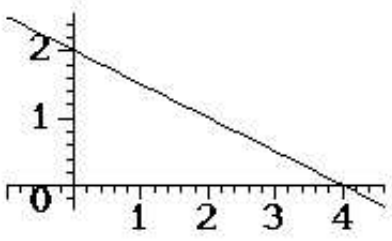
3a). Find a linear map of the plane, $A : \mathbb{R}^2 -> \mathbb{R}^2$ that does the following transformation of the letter **F** (here the smaller **F** is transformed to the larger one):




b). Find a linear map of the plane that inverts this map, that is, it maps the larger **F** to the smaller.

4. (see the graph on the right)
a). If the horizontal axis is $x$ and the vertical axis is $y$, find the equation for $y$ as a function of $x$?
b). If the horizontal axis is $x$ and the vertical axis is $\log y$, find the equation for $y$ as a function of $x$.
c). If the horizontal axis is $\log x$ and the vertical axis is $\log y$, find $y$ as a function of $x$.




5. A multinational company has branches in the US., Japan, and Europe. In 1990, it had assets of \$5 million: \$3 million are in the U.S. and \$2 million in Europe. Each year 1/2 the U.S. money stays home, 1/4 goes to both Japan and Europe. For Japan and Europe, 1/2 stays home and 1/2 is sent to the U.S.
a). Find the transition matrix of this Markov chain.
b). Find the limiting distribution of the \$5 million as the world ends.

6. Say you seek a parabola of the *special form* $y = a + bx^2$ to pass through the three data points $(-1,\, 2)$, $(0,\, 0)$, $(2,\, 3)$.
a). Write the (over-determined) system of equations you would like to solve ideally.
b). Using the method of least squares write the normal equations for the coefficients $a, b$.
c). Explicitly find the coefficients $a$, $b$.

7. Discuss which airline has the better "On Time" record.

| | **Northern Airline** | | | **Travel West** | | |
|---|---|---|---|---|---|---|
| Destination | # arrivals | # on time | % on time | # arrivals | # on time | % on time |
| Phoenix | 200 | 190 | 95% | 5800 | 5220 | 90% |
| Seattle | 2000 | 1700 | 85% | 400 | 300 | 75% |
| **Total** | 2200 | 1890 | 85.9% | 6200 | 5520 | 89% |

[We are seeking a *quantitative* response, not an intuitive one. Since you don't have calculators, approximate calculations will be adequate.]