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Signature Printed Name

Math 210
March 8, 2001

Mid-Term Exam

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1:30 — 2:50

Directions: This exam has 8 problems (*10 points each*). To receive full credit your solution must be clear and correct. No fuzzy reasoning. You have 1 hour 20 minutes. Closed book, no calculators, but you may use one sheet of paper with notes. 30% of your course grade. Please box your answers.

Score	
1	
2	
3	
4	
5	
6	
7	
8	
Total	

1. Describe what both of the following perl scripts will do.

a).
```
#!/usr/bin/perl
$sum=0;
for ($k=1; $k <3; $k++) {
    $sum = $sum + 1/(2*$k);
    }
print "Sum = $sum\n";
```

b).
```
#!/usr/bin/perl
$sum=0;
for ($k=1; $k <3; $k++) {
    $sum = $sum + 1/(2*$k);
    print "Sum = $sum\n";
    }
```

2. The next three players in a game win 30%, 20% and 25% of the time, respectively. What is the likelihood that *none* of them will win this time? [Equivalent wording: It is the fifth inning of a baseball game. The batting averages of the next three batters are .300, .200, and .250. Say they face an average pitcher. What is the likelihood that *none* of them will get a hit this inning?]

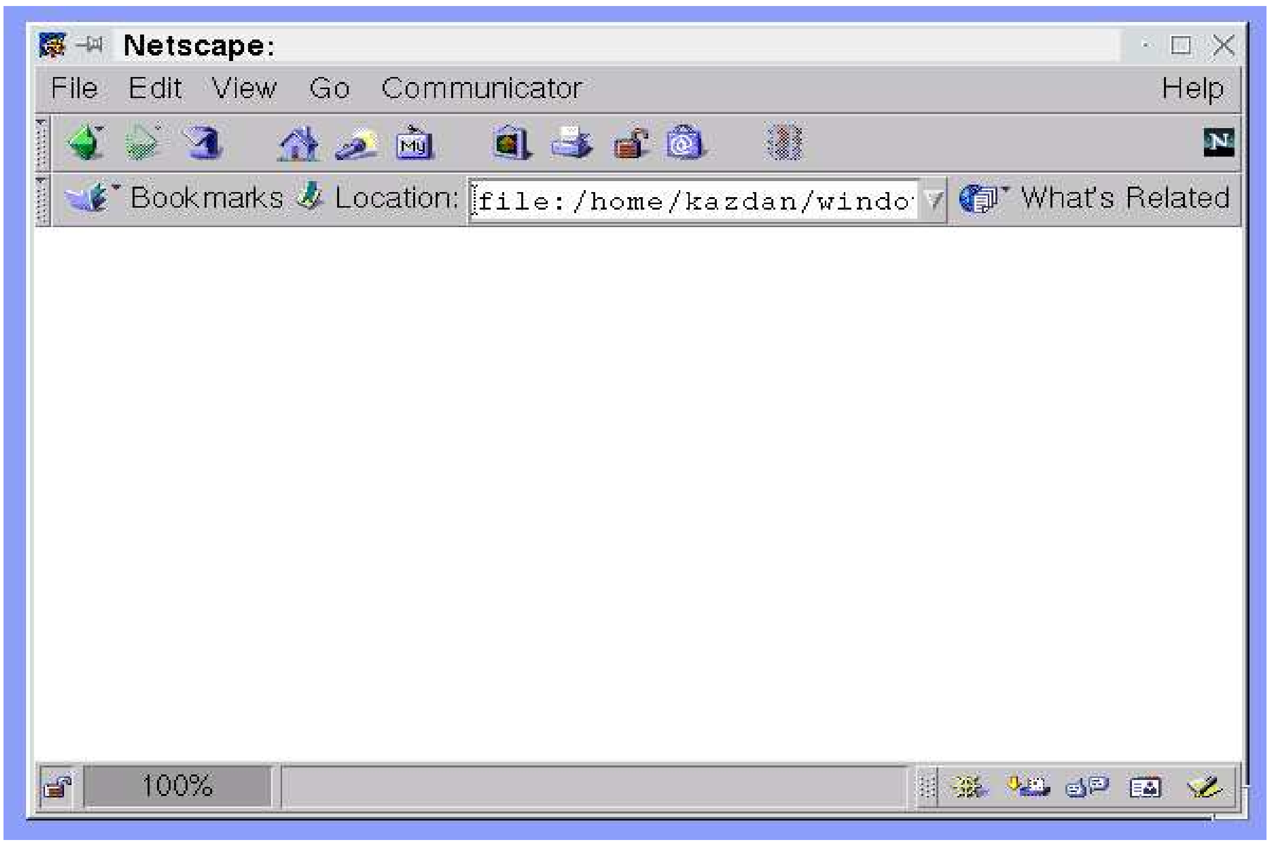
3. The following describes a web page. How will it appear? (fill-in the blank page below).

```
<html><head><title>Spring Break</title></head>
<body bgcolor=yellow>
<center><H1> Spring Break</H1></center>

Spring Break begins this weekend.

Enjoy it.

<br>
    Bye Bye
</body></html>
```



4. A multinational company has branches in the US., Japan, and Europe. In 1990, it had assets of $4 million: $2 million are in the U.S. and $2 million in Europe. Each year 1/2 the U.S. money stays home, $1/4 goes to both Japan and Europe. For Japan and Europe, 1/2 stays home and 1/2 is sent to the U.S.

a). Find the transition matrix of this Markov chain.

b). Find the limiting distribution of the $4 million as the world ends.

5. Say you seek a parabola of the *special form* $y = a + bx^2$ to pass through the three data points $(-1, 2)$, $(0, 1)$, $(2, 3)$.

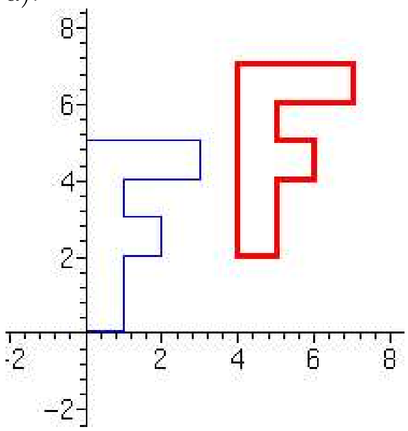
a). Write the (over-determined) system of equations you would like to solve ideally.

b). Using the method of least squares write the normal equations for the coefficients a, b.

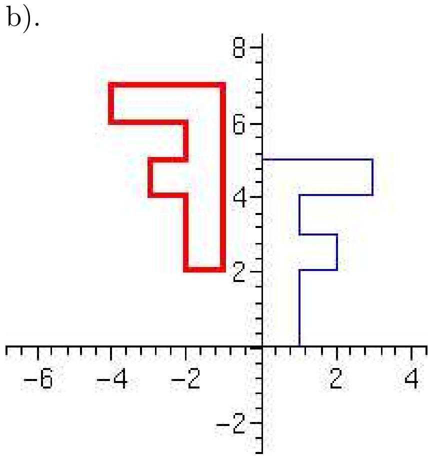
c). Explicitly find the coefficients a, b.

6. For both of the following figures find a matrix that gives the indicated linear transformation.

a).



b).



7. A square matrix P is called a *projection* if $P^2 = P$. while R is called a *reflection* if $R^2 = I$. Say you are given a projection P and define a new matrix R as $R = 2P - I$.
a). Show that R is a reflection, that is, show $R^2 = I$.

b). If the projection P keeps a certain vector V unchanged, so $PV = V$, compute RV.

c). If the projection P "kills" a certain vector W, so $PW = 0$, compute RW.

8. A person tests positive for a relatively rare cancer. He learns it has an incidence of 1% among the general population. Thus, before taking the test, and in the absence of any other evidence, his best estimate of the likelihood of having the cancer is 1 in 100.

> Extensive trials have shown that the reliability of the test is 80%. More precisely, it gives a positive result in 20% of the cases where no cancer is present (*false positive*). Moreover, about 2% of the time the test fails to detect the cancer even though it is present (*false negative*).

Question: Given that he tested positive, what is the probability that he has the cancer?