Math 260 — **Exam 3** — Jerry L. Kazdan

April 12, 2012 — 12:00 – 1:20

DIRECTIONS This exam has two parts. PART A has 4 short answer questions (10 points each, so 40 points) while PART B has 4 traditional problems (15 points each, so 60 points). Total: 100 points. *Neatness counts.*

Closed book, no calculators, computers, ipods, cell phones, etc – but you may use one $3'' \times 5''$ card with notes on both sides.

Part A: Four short answer questions (10 points each, so 40 points).

A–1. Let $f(x) := \int_0^x \left(\int_0^t g(s)\, ds \right) dt$ for $x \ge 0$. Rewrite this as an iterated integral with the order of integration reversed, so one first integrates with respect to t.

For the next 3 problems, $\gamma(t) = (x(t),\, y(t))$, $a \le t \le b$, is a smooth curve in the plane and we consider the line integral $J := \int_\gamma p(x,y)\, dx + q(x,y)\, dy$. Give a proof or counterexample for each of the following.

A–2. If $\gamma(t)$ is a *horizontal* line segment and $p(x,y) = 0$ on this segment, then $J = 0$.

A–3. If $\gamma(t)$ is a *vertical* line segment and $p(x,y) = 0$ on this segment, then $J = 0$.

A–4. If $p(x,y) \ge 0$ and $q(x,y) \ge 0$ on γ, and if in defining γ we know that $dx/dt > 0$ and $dy/dt > 0$, then $J \ge 0$.

Part B: Four traditional problems (15 points each, so 60 points).

B–1. Let $\mathbf{F} = y\mathbf{i} + (3+2x)\mathbf{j} + 2\mathbf{k}$, and $\gamma(t)$ be the straight line from $(0,0,0)$ to $(1,2,-3)$. Compute $\int_\gamma \mathbf{F} \cdot d\mathbf{s}$.

B–2. Let $G(x) := \int_{a(x)}^{b(x)} f(t)\, dt$, where $a(x)$ and $b(x)$ are smooth functions with $a(x) < b(x)$, and $f(x)$ is a continuous function. Compute $dG(x)/dx$.

B–3. Compute $\iint_{\mathbb{R}^2} \frac{dx\, dy}{[4 + (x-y)^2 + (x+2y)^2]^2}$

B–4. Let the surface $S \subset \mathbb{R}^3$ be the graph of $z = g(x, y)$ for $(x,\, y)$ in a region D in the xy-plane.

a) Using the parameters $x = u\,,\ y = v\,,\ z = g(u, v)\,$, derive the formula

$$\text{Area}\,(S) = \iint_D \sqrt{1 + \|\nabla g\|^2}\, dx\, dy.$$

b) Apply this to compute the surface area of the part of the plane $x + 2y + z = 2$ in the first octant $x \ge 0\,,\quad y \ge 0\,,\quad z \ge 0\,.$