Classical Examples of PDEs

Laplace equation:

$$\Delta u := \sum_{j=1}^{3} \frac{\partial^2 u}{\partial x_j^2} = 0 \quad \text{(some write } \Delta u = \nabla \cdot \nabla u = \nabla^2 u\text{)}$$

Poisson equation: $-\Delta u = f(x)$

Helmholtz (or eigenvalue) equation: $-\Delta u = \lambda u$

Transport equation: $\dfrac{\partial u}{\partial t} = \displaystyle\sum_{j=1}^{3} b^j \frac{\partial u}{\partial x_j}$

Heat (or diffusion) equation: $u_t - k\Delta u = 0$

Schrödinger equation $iu_t = -\Delta u + V(x)u$

Wave equation: $u_{tt} - c^2 \Delta u = 0$

Cauchy-Riemann equations: $u_x = v_y, \qquad u_y = -v_x$

Maxwell's equations in a vacuum:

$$\mathbf{E}_t = c\nabla \times \mathbf{H}, \qquad \mathbf{H}_t = -c\nabla \times \mathbf{E}, \qquad \operatorname{div} \mathbf{H} = 0, \quad \operatorname{div} \mathbf{E} = 0$$

Euler's (nonlinear) equations for incompressible inviscid flow:

$$\mathbf{u}_t + \mathbf{u} \cdot \nabla \mathbf{u} = -\nabla p, \qquad \operatorname{div} \mathbf{u} = 0$$