Homework Set 0 [Due: Never]

1. Solve the following system – or show that no solution exists:

$$\begin{aligned} x+2y \quad\quad &= 1 \\ 3x+2y+4z &= 7 \\ -2x+y-2z &= -1 \end{aligned}$$

2. Let $S := \begin{pmatrix} 2 & 5 \\ 1 & 3 \end{pmatrix}$.

   a) Find S^{-1}.

   b) For which constant(s) λ is the matrix $\begin{pmatrix} 2-\lambda & 5 \\ 1 & 3-\lambda \end{pmatrix}$ invertible?

   c) Let $D := \begin{pmatrix} -1 & 0 \\ 0 & 3 \end{pmatrix}$ and $B := SDS^{-1}$. Compute B^{10}.

3. Solve all of the following equations. [Note that the left sides of these equations are identical.]

   a). $\begin{aligned} 2x+5y &= 5 \\ x+3y &= -1 \end{aligned}$ b). $\begin{aligned} 2x+5y &= 0 \\ x+3y &= -2 \end{aligned}$ c). $\begin{aligned} 2x+5y &= 1 \\ x+3y &= 0 \end{aligned}$ d). $\begin{aligned} 2x+5y &= 2 \\ x+3y &= 1 \end{aligned}$

4. Let A and B be 2×2 matrices.

   a) If B is invertible and $AB=0$, show that $A=0$.

   b) Give an example where $AB=0$ but $BA\neq 0$.

Equations as Maps

The next set of problems have an entirely different character. They are about viewing systems of equations as maps. Think of this an an introduction to *computer graphics*. We'll use these ideas throughout Math 312.

The standard technique goes back to Descartes' introduction of coordinates in geometry. Say one has two copies of the plane, the first with coordinates (x_1, x_2), the second with coordinates (y_1, y_2). Then the high school equations

$$x_1 - 0x_2 = y_1 \tag{1}$$

$$x_1 + x_2 = y_2 \tag{2}$$

can be thought of as a mapping from the (x_1, x_2) plane to the (y_1, y_2) plane. For instance, if $x_1 = 1$ and $x_2 = 0$, then $y_1 = 1$ and $y_2 = 1$. Thus the point $(1, 0)$ is mapped to the point $(1, 1)$.

1. a) What are the images of $(0,\, 0)$, $(1,\, 0)$, $(1,\, \pi/4)$, and $(0,\, \pi/4)$?

   b) What is the image of the rectangle with vertices at $(0,\, 0)$, $(1,\, 0)$, $(1,\, \pi/4)$, and $(0,\, \pi/4)$? [Draw a sketch.]

   c) What is the image of the rectangle with vertices at $(1,\, 0)$, $(2,\, 0)$, and $(2,\, \pi/4)$, and $(1,\, \pi/4)$?

2. Next consider the *nonlinear* map from the $(x_1,\, x_2)$ plane to the $(y_1,\, y_2)$ plane

$$
\begin{aligned}
x_1 \cos x_2 &= y_1 \\
x_1 \sin x_2 &= y_2
\end{aligned}
$$

   a) What are the images of $(1,\, 0)$, $(1,\, \pi/4)$, and $(0,\, \pi/4)$?

   b) What is the image of the rectangle with vertices at $(0,\, 0)$, $(1,\, 0)$, $(1,\, \pi/4)$, and $(0,\, \pi/4)$? [Draw a sketch.]

   c) What is the image of the rectangle with vertices at $(1,\, 0)$, $(2,\, 0)$, and $(2,\, \pi/4)$, and $(1,\, \pi/4)$?

3. Consider the linear map from the $(x_1,\, x_2)$ plane to the $(y_1,\, y_2)$ plane defined by the equations

$$
\begin{aligned}
ax_1 + bx_2 &= y_1 \\
cx_1 + dx_2 &= y_2,
\end{aligned}
$$

   where the (real) coefficients satisfy $ad - bc \neq 0$. Show that the image of a straight line ℓ in the $(x_1,\, x_2)$ plane is a straight line in the $(y_1,\, y_2)$ plane, and that this image contains the origin if and only if ℓ contains the origin.

[Last revised: September 2, 2012]