1. Find a point on the surface $x^2 + (y+x)^2 + (z-x)^2 = 25$ where the tangent plane is parallel to the yz-plane.

   A. $(0,5,0)$
   B. $(0,-5,0)$
   C. $(2,2,\frac{1}{5})$
   D. $(5,-5,5)$
   E. $(3,0,0)$
   F. $(2,2,-5)$
   G. $(0,0,3)$
   H. $(0,0,5)$

Normal vector of the tangent plain at (x,y,z) is

$$\left(2x+2(x+y)-2(z-x),\ \ 2(x+y),\ \ 2(z-x)\right)$$

normal vector of the yz-plain is

$$(1,0,0)$$

they need to be parallel, so

$$\begin{cases} 2(x+y)=0 \\ 2(z-x)=0 \end{cases}$$

$\Rightarrow\ y=-x,\quad z=x$

Plug in to the equation of the surface, get

$x^2=25$

$\Rightarrow\ x=\pm 5,\quad y=\mp 5,\quad z=\pm 5$

So there are two points satisfying the requirement:

$(5,-5,5)$ and $(-5,5,-5)$

choose $\boxed{D}$

2. Find the minimum value of the function $f(x, y) = 4 - 6x - 8y$ when subject to the constraint $x^2 + y^2 = 1$.

   A. -18
   B. -6
   C. $-28/5$
   D. $-26/5$
   E. $8/5$
   F. 4
   G. $34/5$
   H. 14

Use Lagrange multiplier:

$$(4 - 6x - 8y) + \lambda(x^2 + y^2 - 1)$$

get

$$\left\{\begin{array}{l} -6 + 2\lambda x = 0 \\ -8 + 2\lambda y = 0 \\ x^2 + y^2 - 1 = 0 \end{array}\right. \Rightarrow \frac{x}{y} = \frac{3}{4} \Rightarrow x = \frac{3}{5},\ y = \frac{4}{5} \text{ or } x = -\frac{3}{5},\ y = -\frac{4}{5}$$

minimum is achieved at $(\frac{3}{5}, \frac{4}{5})$,

$$f(\tfrac{3}{5}, \tfrac{4}{5}) = 4 - \frac{18}{5} - \frac{32}{5} = -6.$$

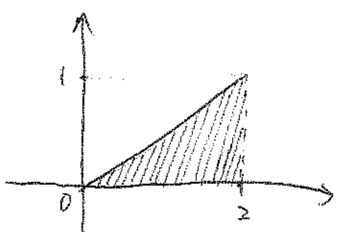
Choose $\boxed{B}$

3. Evaluate :

$$\int_0^1 \int_{2y}^2 4e^{x^2}\,dx\,dy$$

A. 1
B. $-4e^4$
C. $4e^4$
D. e^8
E. $1-e^4$
F. e^4
G. e^4-1
H. 0

Interchange the order of integral



$$\begin{cases} 2y \le x \le 2 \\ 0 \le y \le 1 \end{cases} \Rightarrow \begin{cases} 0 \le y \le \frac{1}{2}x \\ 0 \le x \le 2 \end{cases}$$

$$\int_0^1\int_{2y}^2 4e^{x^2}\,dx\,dy = \int_0^2\int_0^{\frac{1}{2}x} 4e^{x^2}\,dy\,dx$$

$$= \int_0^2 4e^{x^2}\cdot \tfrac{1}{2}x\,dx$$

$$= \int_0^2 2e^{x^2}\,x\,dx$$

$$= \int_0^{2} e^{x^2}\,d(x^2)$$

$$= e^{x^2}\Big|_0^{2}$$

$$= e^4 - e^0$$

$$= e^4 - 1$$

Choose $\boxed{G}$

4. The maximum and minimum values of the function $f(x,y) = x^2 + y^2 + x^2y + 2$ on the square $-1 \le x \le 1, -1 \le y \le 1$ are:

A. $max = 5, min = -2$
B. $max = 2, min = -2$
C. $max = 5, min = -3$
D. $max = 2, min = -2$
E. $max = 5, min = 2$
F. $max = 3, min = 2$
G. $max = 3, min = -2$
H. $max = 5, min = 3$

First find critical points on the interior:

$$\begin{cases} f_x = 0 \\ f_y = 0 \end{cases} \Rightarrow \begin{cases} 2x + 2xy = 0 \\ 2y + x^2 = 0 \end{cases} \Rightarrow \begin{cases} x(1+y) = 0 \\ y = -\frac{1}{2}x^2 \end{cases} \Rightarrow \begin{cases} x=0 \text{ or } y=-1 \\ y = -\frac{1}{2}x^2 \end{cases}$$

$x=0 \Rightarrow y=0$ $\qquad f(0,0)=2$

$y=-1 \Rightarrow x=\pm\sqrt{2}$ out of range

Then find extrema on the boundary:

$x=-1$, $\quad f(-1,y) = 1+y^2+y+2 = y^2+y+3 = (y+\frac{1}{2})^2 + \frac{11}{4}$

$y=-\frac{1}{2}$, $\quad f(-1,-\frac{1}{2}) = \frac{11}{4}$

$y=1$, $\quad f(-1,1)=5$

$y=-1$, $\quad f(-1,-1)=3$

$x=1$: $\quad f(1,y) = y^2+y+3$ same as the previous case

$y=1$: $\quad f(x,1) = 2x^2+3$,

$x=0$, $\quad f(0,1)=3$.

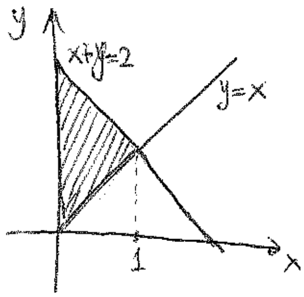
$x=\pm1$, $\quad f(\pm1,1)=5$

$y=-1$, $\quad f(x,-1)=3$.

So $max = 5$, $min = 2$, choose $\boxed{E}$

5. Find the volume of the region that lies under the paraboloid $z = x^2 + y^2$ and above the triangle enclosed by the lines $y = x, x = 0$ and $x + y = 2$ in the xy plane.

   A. 11
   B. $2\pi/3$
   C. 9
   D. π
   E. 1/2
   F. 0
   G. 2/5
   H. 4/3



The lines $y=x$ and $x+y=2$ intersects at $x=1$.

$$\text{Volumn} = \int_0^1 \int_x^{2-x} (x^2+y^2)\, dy\, dx$$

$$= \int_0^1 \left(x^2 y + \frac{1}{3}y^3\right)\Big|_x^{2-x} dx$$

$$= \int_0^1 x^2(2-x) + \frac{1}{3}(2-x)^3 - x^3 - \frac{1}{3}x^3\, dx$$

$$= \int_0^1 2x^2 - x^3 + \frac{1}{3}(2-x)^3 - x^3 - \frac{1}{3}x^3\, dx$$

$$= \left(\frac{2}{3}x^3 - \frac{1}{4}x^4 - \frac{1}{12}(2-x)^4 - \frac{1}{4}x^4 - \frac{1}{12}x^4\right)\Big|_0^1$$

$$= \frac{2}{3} - \frac{1}{4} - \frac{1}{12} - \frac{1}{4} - \frac{1}{12} + \frac{16}{12}$$

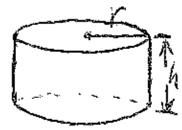
$$= \frac{4}{3}$$

Choose $\boxed{H}$

6. A cylindrical tin can with a bottom but ***no*** top is to be built out of 12π square inches of tin. What is the maximum volume of such a can?

   A. 4π cubic inches
   B. 8π cubic inches
   C. 12π cubic inches
   D. 16π cubic inches
   E. 9π cubic inches
   F. 18π cubic inches
   G. 27π cubic inches
   H. 32π cubic inches

Suppose radius is r
and height is h



We want to maximize $Vol = \pi r^2 h$
under the constraint: $\pi r^2 + 2\pi r h = 12\pi$

From the constraint, we get

$$r^2 + 2rh = 12$$

So $$h = \frac{12 - r^2}{2r}$$

Plug into $Vol = \pi r^2 h$

get $$Vol = \pi r^2 \cdot \frac{12 - r^2}{2r} = \frac{1}{2}\pi r(12 - r^2)$$

$Vol'(r) = 0 \Rightarrow 12 - 3r^2 = 0 \Rightarrow r = 2, \ h = 2$

Maximum volume is 8π

So choose $\boxed{B}$

7. Let $f(x, y) = y^2 e^x$. Use the differential to approximate $(3.05)^2 e^{0.1}$.

A. 10.2
B. 1.2
C. 10.05
D. 1.05
E. 4.2
F. 4.05
G. 10.1
H. 10.645

$$f(x+\Delta x, y+\Delta y) \approx f(x,y) + f_x(x,y)\,\Delta x + f_y(x,y)\,\Delta y$$

choose $x=0$, $y=3$, $\Delta x = 0.1$, $\Delta y = 0.05$.

$f(x,y) = y^2 e^x \Rightarrow f(0,3) = 9$

$f_x(x,y) = y^2 e^x \Rightarrow f_x(0,3) = 9$

$f_y(x,y) = 2ye^x \Rightarrow f_y(0,3) = 6$

So $f(0.01, 3.05) \approx 9 + 9 \cdot 0.1 + 6 \cdot 0.05$

$= 9 + 0.9 + 0.3$

$= 10.2$

Choose $\boxed{A}$

8. Consider the function $f(x, y) = y^2 + 2xy - x^2 + 2x - 10y$. Find its critical points and determine their type.

A. Local max at $(0, 0)$.
B. Local min at $(0, 0)$.
C. Saddle point at $(0, 0)$.
D. Local max at $(3, 2)$.
E. Local min at $(3, 2)$.
F. Saddle point at $(3, 2)$.
G. There are no critical points .
H. There is more than one critical point.

$$\begin{cases} f_x = 2y - 2x + 2 = 0 \\ f_y = 2y + 2x - 10 = 0 \end{cases} \Rightarrow \begin{cases} x = 3 \\ y = 2 \end{cases}$$

critical point is $(3, 2)$.

$f_{xx} = -2, \quad f_{yy} = 2, \quad f_{xy} = 2,$

$\det\begin{pmatrix} -2 & 2 \\ 2 & 2 \end{pmatrix} = -8 < 0$

Hessian is indefinite $\Rightarrow$ saddle point.

Choose $\boxed{F}$.

9. Consider the function $f(u,v) = uv + e^{v-1} - cos(u)$ where u and v are functions of x and y with the properties:
$u(2,-1) = 0, \quad v(2,-1) = 1,$
$\frac{\partial u}{\partial x}(2,-1) = 2, \quad \frac{\partial u}{\partial y}(2,-1) = 3,$
$\frac{\partial v}{\partial x}(2,-1) = -5, \quad \frac{\partial v}{\partial y}(2,-1) = 7.$
Then $\frac{\partial f}{\partial x}$ at the point $(x,y) = (2,-1)$ is:
A. 0
B. 1
C. -1
D. $e^{-1} - 1$
E. $-2 + e^{-2} - cos(2)$
F. -3
G. 9
H. 8.

According to chain rule

$$\frac{\partial f}{\partial x} = \frac{\partial f}{\partial u}\frac{\partial u}{\partial x} + \frac{\partial f}{\partial v}\frac{\partial v}{\partial x}$$

$$= (v + \sin u)\frac{\partial u}{\partial x} + (u + e^{v-1})\frac{\partial v}{\partial x}$$

plug in

$u(2,-1) = 0$

$v(2,-1) = 1$

$\frac{\partial u}{\partial x}(2,-1) = 2$

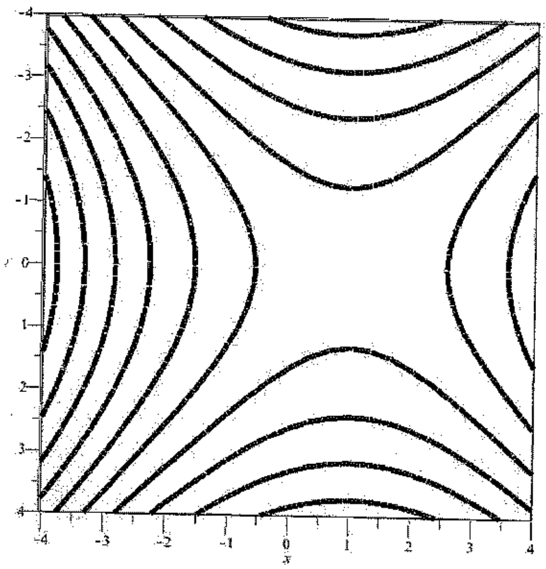
$\frac{\partial v}{\partial x}(2,-1) = -5$

get $\frac{\partial f}{\partial x}(2,-1) = (1 + \sin 0)\cdot 2 + (0 + e^{1-1})\cdot(-5)$

$= 2 - 5$

$= -3$

choose $\boxed{F}$



10. Above you will find a graph of the level curves of a function $z = f(x, y)$. Then $f(x, y) =$

A. $(x-1)^2 + y^2$
B. $(x+1)^2 + y^2$
C. $(x-1)^2 - y^2$
D. $(x+1)^2 - y^2$
E. $x^2 - (y-1)^2$
F. $x^2 + (y+1)^2$
G. $(x-1)y$
H. $(x+1)y$

The graph is for hyperbolas centering at $(1, 0)$.

So choose $\boxed{C}$