RESEARCH STATEMENT

LARS KADISON

1. List of 10 notable publications and summary of my contribution

(1) *Hopf subalgebras and tensor products of generalized permutation modules, J. Pure Appl. Algebra 218 (2014), 367–380.* [13] In this article, subgroup depth and its generalizations to Hopf subalgebras are shown to be determined by an algebraic module.

(2) (with S. Burciu) *Subgroups of depth three, Surv. Diff. Geom. 15 (2011), 17–36.* In this article in a proceedings with some august company, I determined the matrix inequality relation for depth from the method in paper (4) below.

(3) (with S. Burciu and B. Külshammer) *On subgroup depth, International Electronic J. Algebra 9 (2011), 133–166.* In this article, subgroup depth is introduced for algebraically closed fields of characteristic zero with conditions for reading the depth off of bicolored graphs. I have contributed with the matrix inequality condition for depth and some consequences of this.

(4) *Finite depth and Jacobson-Bourbaki correspondence, J. Pure and Applied Algebra 212 (2008), 1822–1839.* In this article, finite depth is definable in a way that covers operator algebras via my definition of depth three tower of three algebras.

(5) *Infinite index subalgebras of depth two, Proc.A.M.S. 136 (2008), 1523–1532.* In this article, a theorem characterizes general Galois extensions via coaction of bialgebroids in terms of an infinite version of the depth two condition, in a constructive way originating in paper (7) below.

(6) *An approach to quasi-Hopf algebras via Frobenius coordinates, J. Algebra 295 (2006), 27–43.* In this article, Radford's formula for the fourth power of the antipode is generalized to Frobenius algebras with anti-automorphism and it is shown how to obtain various variant formulas for weak Hopf algebras, quasi-Hopf algebras and the Radford formula itself. The paper continues my three papers with A.A. Stolin in Contemporary Mathematics, Beitraege and Journal of Pure and Applied Algebra.

(7) (with K. Szlachanyi) *Bialgebroid actions on depth two extensions and duality, Advances in Math. 179 (2003), 75-121.* In this paper, I have checked that all aspects of the bicategorical theory of Szlachanyi work and made adjustments, for ring extensions that model depth two Frobenius extensions in paper (8) below. My experience with H-separable extensions in several other papers guided my work.

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(8) (with D. Nikshych) *Hopf algebra actions on strongly separable extensions of depth two, Adv. in Math. 163 (2001), 258-286.* [12] In this paper, I have provided the necessary Frobenius extension theory and Hopf-Galois theory to make calculations in operator algebras of Nikshych-Vainerman work.

(9) (with D. Nikshych) *Frobenius extensions and weak Hopf algebras, J. Algebra 244 (2001), 312-342.* In this paper, I performed a similar function as in paper (8), and I gained a solid foundation in the new field of weak Hopf algebras of mathematical physics.

(10) *The Jones polynomial and certain separable Frobenius extensions, Journal of Algebra 186 (1996), 461–475.* This paper shows one way to simplify algebraically the method of Jones in finding characters for the braid groups that make the Markov moves in link theory. This method received some attention from link homologists starting with Khovanov.

2. Synopsis of the CV (in third person)

Kadison received his Ph.D. in 1989 from University of California at Berkeley with a dissertation in cyclic homology and algebraic K-theory. Kadison is listed with 53 publications and 406 citations by about 170 authors on MathSciNet. The monograph on Frobenius extensions by Kadison tops the list with more than 76 citations, followed by the article, Bialgebroid actions on depth two extensions and duality (with Szlachanyi) in Adv. Math. in 2003, with more than 51 citations. Kadison has more than 34 citations of joint work with Nikshych, followed by 32 citations of joint work with Stolin. The recent publication, On Subgroup Depth (with Külshammer and Burciu, 2011, [5]), has more than 18 citations, several more on the arXiv and is the cornerstone article of a new area of modular and ordinary group representation theory, subgroup depth.

Kadison has worked for in all six years at the assistant, associate and full levels of professorship at Universities in Denmark, Norway and Louisiana, respectively. He has been a teaching visitor at top 25 universities in Pennsylvania and California. He has been employed since 2010 as Investigador at the Faculty of Science at the University of Porto (as a five year research professor, one of circa 200 successful applications in an all-science NSF competition within Portugal, with a year's extension). He has supervised a Scottish Ph.D. student to a completed thesis in 2014, and another from Costa Rica to a completed thesis on September 30, 2016. Kadison has taught several graduate courses in Porto and given several seminar talks there as well as conference talks in Sweden, France and Belgium.

3. Synopsis of the research project and career development plan

One of the themes of Feit's book on group representation theory is that of an algebraic module, and that a permutation module is algebraic (as an isoclass in Green ring). Thus a finite group has finite depth over any subgroup,and it is natural to ask if this is true for Hopf subalgebras, a question raised by Boltje et al in [3, 2010]. Since algebraic elements make sense in noncommutative rings as well, the question may be rephrased as, is the quotient module Q of any Hopf subalgebra $R \subseteq H$ algebraic (in the representation ring or Green ring of either Hopf algebra)? I have made a preliminary investigation in the last two years of this type of question, in which the equivalence of this question with several others is shown, including a reformulation into depth of a Hopf algebra H in its smash product $Q * \# H$, a thesis topic of one of my Ph.D. students. Now the Green ring of Hopf algebras is an area of active investigation by Chen, van Oystaeyen et al, Cibils and others, and their results have a bearing on such a question; e.g. if the Green ring is commutative, then algebraic modules form a subring. Thus results on simple modules being algebraic such as Berger's for solvable groups are applicable for certain quantum groups and their grouplike subalgebras.

4. Synopsis of basic terminology in application

The Green ring or representation ring of a group algebra or Hopf algebra is the ring of isomorphism classes of finite-dimensional modules under direct sum and tensor product (using the diagonal action or coproduct). If the Hopf algebra is semisimple, it is a ring structure on the Grothendieck group, also isomorphic to the character ring. There are many classical results on the Green ring recorded in Feit, Landrock and others; and recently there are results on the Green ring of Hopf algebras, such as by van Oystaeyen et al showing the noncocommutative Taft algebras have commutative Green rings.

For a finite-dimensional Hopf algebra H over an algebraically closed field the module category H-Mod is a finite tensor category studied by Etingof and Ostrik. An algebra in the tensor category of comodules is a comodule algebra A (with multiplication and unit being H-comodule morphisms), and A-Mod is a module category over H-mod since there is a tensor from A-Mod tensor H-mod into A-Mod (Ostrik, Etingof, Andruskiewicz, et al). Similary there is a notion of H-module coalgebra C, a coalgebra in the category H-Mod. The Doi-Koppinen entwining of A tensor C makes it an A-coring, logically equivalent notions in a sense, where A-coring is a generalization of coalgebra to noncommutative base rings. The depth of an A-coring is a notion comparing tensor powers of the underlying A-bimodule; it is 2n+1 if the n'th tensor power is similar as bimodules to the n+1'st. (Similarity is a notion that in the presence of unique factorization into indecomposables says that the two similar objects have the same constituents.) It is well-defined natural number as the minimum depth of a coring, a ring extension (consider a fundamental coring of the ring extension on its tensor-square), and also a Doi-Koppinen entwining. It has variants called minimum odd, even depth, and h-depth (all closely related with minimums in an interval of 2, if any one is finite). For example, consider H itself and R a coideal subalgebra, we may take Q the quotient module (a technical notion that adapts cosets to Hopf algebras) the epimorphic image H-module coalgebra of H itself; the entwining of H and Q has H-coring depth equal to the subalgebra h-depth of R in H, since one shows the coring to be Galois, equivalently H is a Q-coalgebra-Galois extension of R. In case H is a group algebra, R a subgroup algebra, Q then is a permutation module of cosets, and a cancellation with the unit module in the finite tensor category computes subgroup depth in linear relation with a depth of a G-module, defined by a degree of a minimum polynomial satisfied in the Green ring by the isoclass of Q.

5. Research plan and methods, the major scientific question I wish to address and the objectives of my project

One of the themes of Feit's book on group representation theory is that of an algebraic module; e.g., permutation modules are algebraic as a consequence of Mackey's Theorem. In my paper [13] it is shown that a Hopf subalgebra has finite depth if and only if its quotient module Q (a permutation module of cosets in case one is dealing with a group algebra extension) is algebraic. Thus a finite group has finite depth over any subgroup (with precise upper bounds obtained in [3], and it is natural to ask if this is true for Hopf subalgebras, a question raised in [3]. Since algebraic elements make sense in noncommutative rings as well, the question becomes, is the quotient module Q of any Hopf subalgebra $R \subseteq H$ (where H is finite-dimensional) algebraic? I have made a preliminary investigation in the last two years of this type of question, sorting out technical issues as well as showing the equivalence of this question with several others, including a reformulation into depth of a Hopf algebra H in its smash product $Q * \# H$, a thesis topic of my ph.d. student, Christopher Young. The computation of Green rings of Hopf algebras is an area of active investigation by van Oystaeyen et al, and their results have a bearing on such a question; e.g. if the Green ring is commutative, then algebraic modules form a subring. Thus results on simple modules being algebraic such as Berger's for solvable groups are applicable for certain quantum groups and their grouplike subalgebras.

Another approach is to view Q as the induced module from the counit one-dimensional representation of the subalgebra R. Thus Q represents an element in the ideal $A(H,R)$ of relative projectives in the complex Green ring $A(H)$. How much modular representation theory in [15, Landrock] generalizes to Hopf algebra pairs? When is the ideal $A(H,R)$ finite-dimensional? This is the case when either H or R has finite representation type. It would be interesting to obtain more information on this question; in particular, for such Hopf algebra extensions of "finite representation type" (where $A(H,R)$ is finite-dimensional) the element $[Q]$ is necessarily algebraic.

Answering the question of whether Q is always algebraic will have at least two approaches. One approach is to look for a counterexample. I have shown in [13] that both Hopf algebras R and H must be of infinite representation type (tame or wild) for this to happen. For example, the small quantum group $U_q(sl_2)$ has infinite representation type for q a primitive n'th root-of-unity where $n > 2$, but the Taft algebras mentioned before are Nakayama algebras and have finite representation type. (Already here we have Hopf subalgebras with Q not semisimple [13].) The quantum groups of the other simple Lie algebras mostly have wild type [14]. The other approach is to attempt a proof along the lines of Mackey for Hopf algebras or classes of Hopf algebras such as quantum groups: finding tensor product theorems for classes of modules that include Q. There may be an approach to this problem with a heavy use of coalgebra theory, in which case I will have to school myself in these techniques and likely join forces with a coauthor. One area of coalgebra theory that already is closely related to depth is the wedge product application in the paper [6], in which a maximal nilpotent Hopf ideal is defined in terms of stabilization of wedge powers of the radical ideal of a Hopf algebra. The least power is related to the depth of a module whose tensor powers eventually are faithful [11]. This approach to depth has yielded for my team some results on at what power the adjoint representation of a group G on itself is faithful [16] in terms of a group algebra within its Drinfeld double D(G).

A long-term project that is not unrelated to the one just treated is to attempt an explanation of subgroup depth only taking observed values (using a GAP subprogram [BKK]) on the odd integers and $\{2, 4, 6\}$. The (minimum subgroup) depth of the permutation group S_n in S_{n+1} is 2n-1, normal subgroups have depth 2, the dihedral group on a square in S_4 has depth 4, and the 108 element subgroup of a classical projective geometric group of 432 elements has depth 6, Horvath has found a subgroup of order 1 437 004 800 in the alternation group A_{15} of depth 8, but 10, 12 and evens beyond have not been observed as values of subgroup minimum depth. Perhaps this has an explanation along the lines of Gabriel's theorem (on which path algebras of acyclic quivers have finite representation type, using Dynkin diagrams) and Artin's classification of the simple Lie algebras (again using Dynkin diagrams). It is perhaps a consequence of bicolored graphs and what shape they may take as complex subgroup algebra inclusions. More generally one may ask what values are assumed by minimum depth of finite Hopf algebra extensions, or even coalgebra-Galois extensions, which include all coideal subalgebra-Hopf algebra pairs [4].

Related to this long-term project are a few that are perhaps more short-range. A subgroup of h-depth 1 is necessarily the whole group, since h-depth 1 for a ring extension is H-separability, a strong condition for both group algebra and Hopf algebra extensions [13]. In this case h-depth 1 implies depth 1. Having depth 2 is normality (or ad-stability) for a Hopf subalgebra [2]. Studying various examples one conjectures that for a subgroup pair $H \leq G$, the minimum h-depth (an odd integer $d_h(H,G)$) and minimum depth (any natural number d(H,G)) are related by the inequality, $d(H,G) \leq d_h(H,G)$. Another project is recently completed, to show why Danz's result in [9] for minimum depth of symmetric group algebra extensions with twisted coefficients in the complex numbers is less than the value 2n-1 (for corresponding n) mentioned above; the difference goes to infinity with n: see my published paper [10], which makes use of a depth of a Galois coring defined by the Doi-Koppinen entwined structure of a crossed product of group and subgroup on the one hand, both comodule algebras, and on the other hand, the quotient (coalgebra) permutation module, which has depth linearly related to the subgroup depth. As a Galois coring its depth is equal to the h-depth of the crossed product extension, in particular a result for twisted group algebra extensions. This method also works for smash or cross product of a left H-module algebra with both a Hopf algebra H and a Hopf subalgebra R: the depths satisfy $d_h(A\#R, A\#H) \leq d_h(R,H)$.

6. The expected outcomes of my research project

I think most mathematicians would have some degree of expectation that Hopf subalgebras would behave like group algebra extensions with regard to finite depth, equivalently having algebraic quotient module. Proving this with a Mackey-like theorem would be a very good piece of mathematics that would boost the state of knowledge of Hopf algebras and their representations. If on the other hand a counterexample is found among Hopf subalgebra pairs of infinite representation type, I think this would cause a small sensation that would set even more so Hopf algebras apart from group algebras. The ten or fifteen years left in my career would benefit from a boost coming from either outcome, or even mixed outcomes, such as finding a counterexample and proving that Q is algebraic for most other Hopf algebra pairs. I have co-organized with A.Stolin a special session on Quantum Groups and Representation Theory at the Joint Meeting of the A.M.S. with the E.M.S. and S.P.M. in Porto, Portugal in June 2015. We had speakers from the

U.S.A., Canada, Germany and Sweden in 6 hours of parallel session talks, including several talks on algebraic depth.

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