Problem 1: Suppose for contradiction that $C$ has an order relation $\leq$ that makes $C$ into an ordered field. In that case the axioms imply that $-1 < 0$, but also that $x^2 \geq 0$ for all $x \in C$. This is a contradiction since we would have

$$-1 < 0 \leq (i)^2 = -1.$$ 

Problem 2: Let $(M, d)$ be a finite metric space, and write $M = \{m_1, \ldots, m_n\}$. We define $\psi : M \rightarrow \mathbb{R}^n$ by $\psi(m) = (d(m, m_1), \ldots, d(m, m_n))$ for all $m \in M$.

Claim: $\|\psi(m) - \psi(m')\|_\infty = d(m, m')$ for all $m, m' \in M$.

Let's prove it in two parts. Let $m, m' \in M$.

"$\leq$": For any $i = 1, \ldots, n$ we have two triangle inequalities:

\[
\begin{align*}
(d(m_i, m) + d(m, m')) &\geq d(m_i, m') \\
(d(m_i, m') + d(m', m)) &\geq d(m_i, m')
\end{align*}
\]

These together imply

$$-d(m, m') \leq d(m_i, m) - d(m_i, m') \leq d(m, m')$$

$$\Rightarrow \quad \|d(m_i, m) - d(m_i, m')\|_\infty \leq d(m, m') \quad \text{for all } i = 1, \ldots, n$$

so

$$\|\psi(m) - \psi(m')\|_\infty = \max_{i = 1, \ldots, n} \|d(m_i, m) - d(m_i, m')\|_\infty \leq d(m, m')$$

"$\geq$": Note that $m' \in \{m_1, \ldots, m_n\}$, so one of the $m_i = m'$.

That means one of the coordinates in $\psi(m') - \psi(m)$ is $d(m', m) - d(m, m') = -d(m, m')$, so the sup-norm is at least \(\|\psi(m') - \psi(m)\|_\infty \geq d(m', m)\). Thus

$$\|\psi(m') - \psi(m)\|_\infty \geq d(m', m).$$
Pg 63 # 3) Orthogonal complement is the set of vectors \( v = (x, y, z) \) that are orthogonal to both given vectors:
\[
\langle v, (3, 2, 2) \rangle = 3x + 2y + 2z = 0
\]
\[
\langle v, (0, 1, 0) \rangle = y = 0
\]
\[
\Rightarrow \begin{cases} 3x = -2z \\ y = 0 \end{cases} \Rightarrow (x, y, z) = t \left(-\frac{2}{3}, 0, 1\right) \text{ for some } t \in \mathbb{R}.
\]
Orth. compl. is \( \{ \left(-\frac{2}{3}, 0, t\right) \mid t \in \mathbb{R} \} \).

Pg 70 # 1) Sup norm:
\[
\|f - g\|_\infty = \sup \{ |f(x) - g(x)| \mid x \in [0, 1] \} = \sup \{ |1-x| \mid x \in [0, 1] \} = 1
\]
\( L_2 \)-norm:
\[
\|f - g\|_2^2 = \int_0^1 (f(x) - g(x))^2 dx = \int_0^1 (1-x)^2 dx = \frac{1}{3}
\]
\[
\|f - g\|_2 = \frac{1}{\sqrt{3}}.
\]
# 3) \[
\langle f, g \rangle = \int_0^1 x \cdot dx = \frac{1}{2}, \quad \langle f, f \rangle = \int_0^1 1 \cdot dx = 1, \quad \langle g, g \rangle = \int_0^1 x \cdot dx = \frac{1}{3}
\]
Since \( \frac{1}{2} \leq \sqrt{\frac{1}{3}} \), Cauchy inequality holds.

0.5 0.5??

Pg 78 # 3) No: e.g. \( z = 1 + i \), \( w = i \) \( \Rightarrow zw = -1 - i \)
\( \text{Re} = -1, \quad \text{Re} = 0, \quad \text{Re}(zw) = -1 \).

# 9) Since \(|z| = n \cdot |z^n|\), it converges to 0 if \(|z| < 1\),
and diverges if \(|z| > 1\), (because then \(|z^n| \to \infty \) as \( n \to \infty \)).