What Logic is Good for CS ?
or
"The Drinking Philosophers" in the Mirror of Horn Linear Logic

to be dwelt upon at length

June 8, 2000

LOGIC, n. The art of thinking and reasoning in strict accordance with the limitations and incapacities of the human misunderstanding.

The basic of logic is the syllogism, consisting of a major and a minor premise and a conclusion – thus:

Major Premise: "60 men can do a piece of work 60 times as quickly as 1 man."

Minor Premise: "1 man can dig a posthole in 60 seconds;"

therefore –
Conclusion: "60 men can dig a posthole in 1 second."

Ambrose Bierce, "The Devil's Dictionary"

1 Classics

A number of *processes*, which need *resources*, may interact with each other in such a way that at any moment only one of them is engaged in a "critical section".

[Dijkstra 1968]: The Dining Philosophers.

Five philosophers who do nothing but eat and think are seated at a round table with a fish in the center of the table. Between each pair of the philosophers is a single fork. A philosopher needs to have the two adjacent forks to eat fish.

How should the philosophers share the forks so that all are allowed to eat as much as they need and that none will starve ?

[Chandy and Misra 1984]: The Drinking Philosophers.

The initial graph is arbitrary. (Big Bang).

In order to mix a drink, a philosopher needs some bottles (not necessarily all).

In addition to that, here:

(a) The "communication topology" may be changed during the course of actions and events.

(b) Quantitative time constraints are imposed on trajectories.

(c) Furthemore, the numerical bounds may be modified by the intermediate actions.

2 A Fish Party

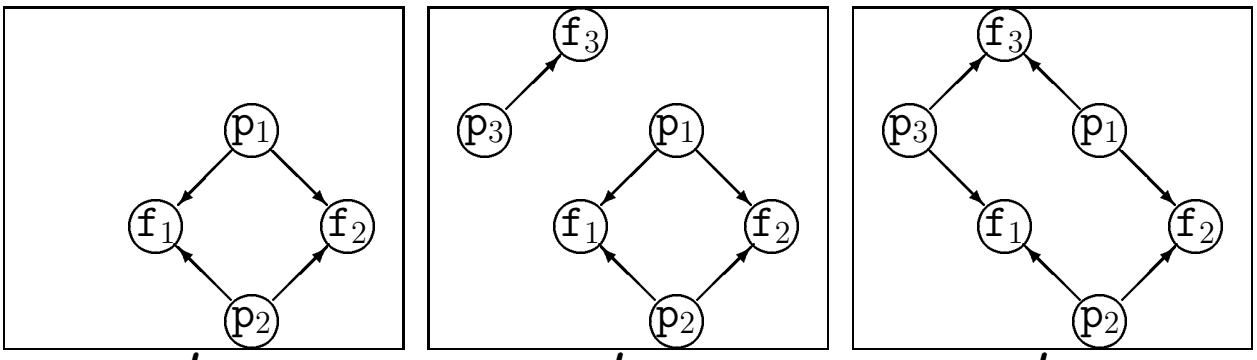
Dr. Fisher has invited his philosophical friends to a fish party BYOF (bring-your-own-fork).

The philosophers are to be seated at a round table, with each pair of the philosophers sharing one fork.

A philosopher needs to have the two adjacent forks to eat fish.

It takes at least E min. to finish a piece of the fish after he starts eating.

Each of the newcomers takes a seat by wedging himself between two adjacent philosophers already seated there, and putting his own fork on the appropriate place on the table.



t_0 t_1 t_2

How should the philosophers share the forks so that all are allowed to eat as much as they need ?

3 Semantics: Trajectories and Scenarios

A *path (trajectory, realization)* $\mathcal{F}$**, a mapping**

$$\mathcal{F} : \texttt{Time} \mapsto \texttt{STATE},$$

shows a possible course of events in the real-time system.

Due to conservation of energy, *only finitely many* **"quantum leaps" may happen within a bounded time interval.**

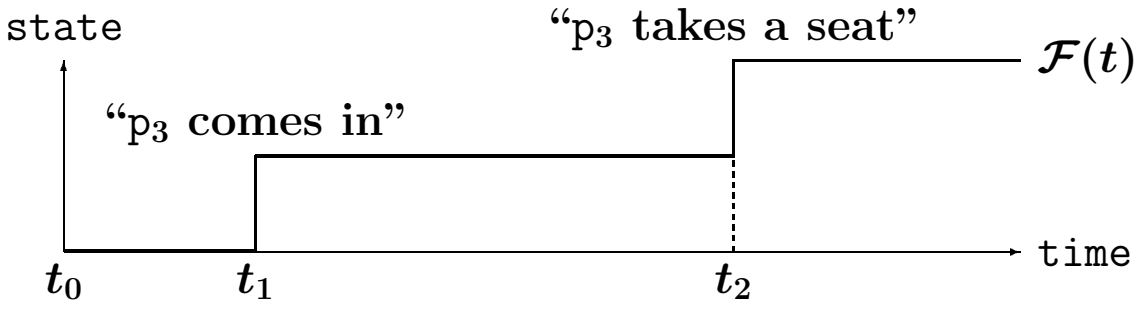
Therefore, $\mathcal{F}$ **is to be** *piecewise continuous*.

Furthermore, $\mathcal{F}$ **is to be a** *piecewise constant function*, **a.k.a. a** *step function*, **whenever** `STATE` **is finite.**

A *scenario* $\mathcal{S}$ **associated with** $\mathcal{F}$ **is a chain of events**

$$e_0, e_1, e_2, e_3, \ldots, e_n, \ldots$$

such that e_n **describes the quantum leap of** $\mathcal{F}$ **at its** n**-th discontinuity point** t_n**,** (t_0 **is the initial point).**

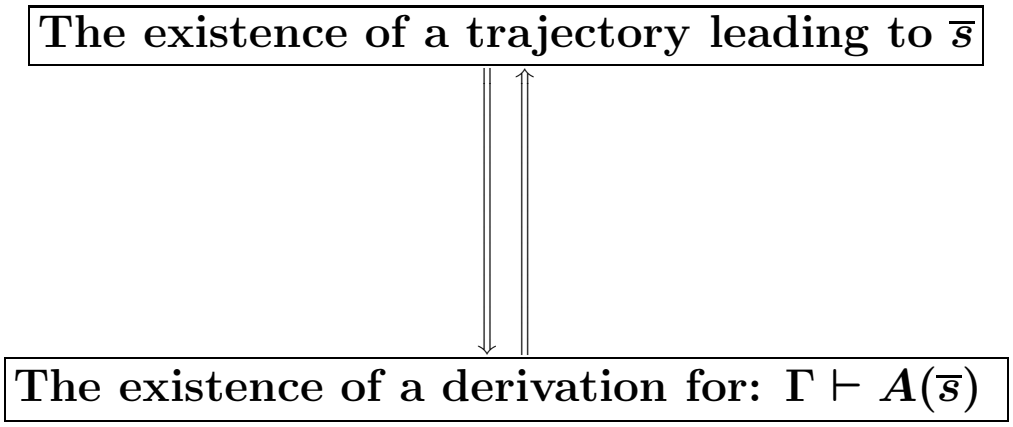


A trajectory $\mathcal{F}$ **is** legal **iff** $\mathcal{F}$ **satisfies all quantitative time constraints for some scenario** $\mathcal{S}$ **associated with** $\mathcal{F}$.

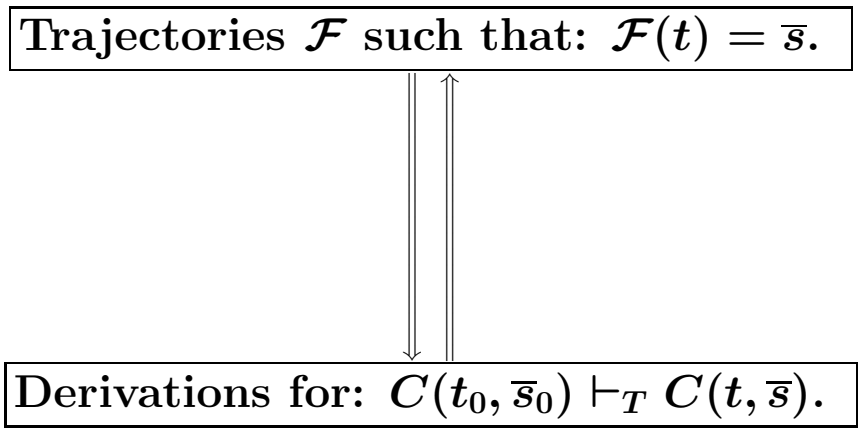
NOTE: "$\mathcal{F} \equiv_{\mathcal{S}} \mathcal{G}$**" respects translation:** $\texttt{time} \mapsto \texttt{time} + \alpha$**, which is responsible for conservation of energy.**

4 Real-Time Systems: Adequacy is the Challenge

The pure existence:



A logical ideal: The full adequacy:



5 Real-Time Systems: Obstructions to Logic

(a) Real time:
A global continuous measurable quantity `time` is assumed in which events occur in irreversible succession from the past through the present to the future: from $-\infty$ to $+\infty$.

$\Longrightarrow$ **A super-complicated set of trajectories.**

(b) Potentially unbounded number of actors.

(c) Dynamically configured topology of actors.

$\Longrightarrow$ **Beyond the finite automaton paradigm.**

(d) Events with global quantitatively delayed effects.

(e) Global quantitative time constraints.

$\Longrightarrow$ **Beyond the Markov processes paradigm.**

(f) Instant local events.

$\Longrightarrow$ **Naïve Horn axioms in the user terms.**

6 The Fish Party in Formal Terms

We introduce the following many-sorted predicates:

(i) $\texttt{Time}(t)$ **:=** "`time` **is** t **(on the global clock)".**

(ii) $E(p, f)$ **:=** "p **is allocated to** f".

(iii) $\texttt{P}(p, s)$ **:=** "**Philosopher** p **is in state** s",
the domain of s **is:** {`new`, `eat`, `idle`},

(iv) $\texttt{F}(f, d)$ **:=** "**Fork** f **is in state** d".
the domain of d **is:** {`ready`, `busy`},

An event is a quantum leap at t**:** $\texttt{Pre}(t, \overline{s}) \vdash \texttt{Post}(t, \overline{s'})$.

The intended meaning of logical connectives is as follows:

(i) $A \otimes B$ **:=** "A **and** B **co-exist**".

$$\frac{A_1 \vdash B_1 \qquad A_2 \vdash B_2}{A_1 \otimes A_2 \vdash B_1 \otimes B_2}$$

(ii) $\exists x\, A(x)$ **:=** "**there appears** x **such that** $A(x)$".

$$A(t) \vdash \exists x\, A(x)$$

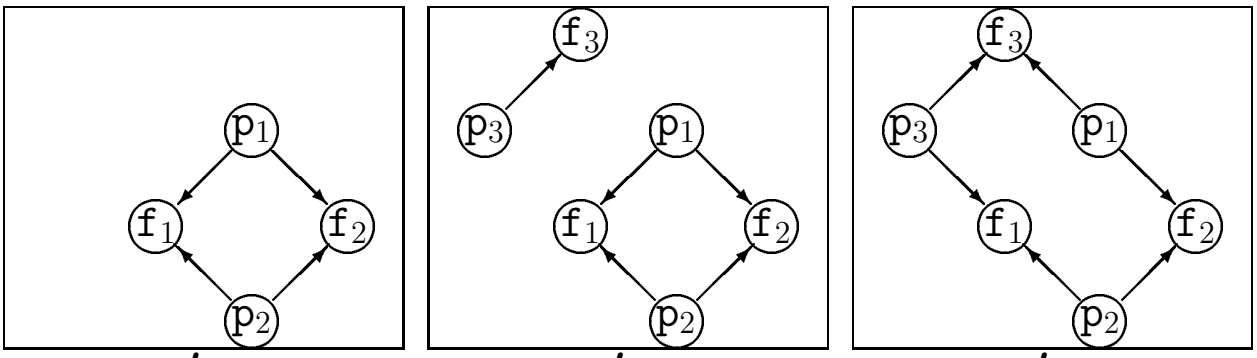
(iii) $A \Rightarrow B$ **represents a function from** A **into** B.

$$A,\ (A \Rightarrow B) \vdash B$$

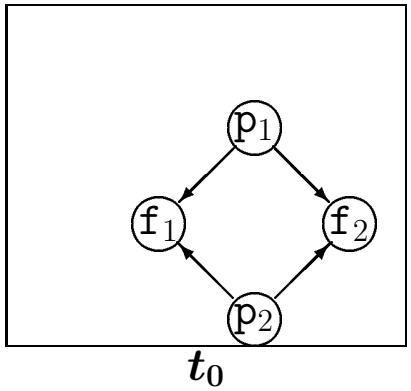
(iv) $A \vdash B$ **:=** "A **can be transformed into** B".

$$\frac{A \vdash B \qquad B \vdash C}{A \vdash C}$$

7 Configurations



The *initial configuration* $\mathtt{C_0}$**:**



is formalized as:

$$[\mathtt{Time}(t_0) \otimes \bigotimes_{i=1,2} \mathtt{P}(\mathtt{p}_i, \mathtt{idle}) \otimes \bigotimes_{i,j=1,2} E(\mathtt{p}_i, \mathtt{f}_j) \otimes \bigotimes_{j=1,2} \mathtt{F}(\mathtt{f}_j, \mathtt{ready})].$$

8 A New Guest Appears: Act 1

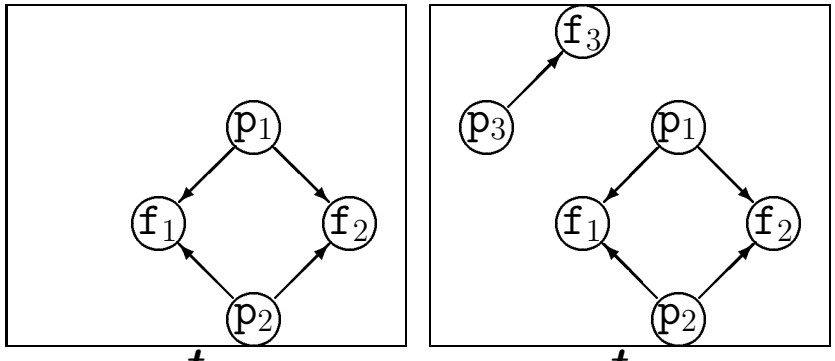
The event:

> **"A new guest comes into the existence (with his own fork)",**

is axiomatized as:

Axiom NEW:

$$\texttt{Time}(\boldsymbol{t}) \vdash \exists \boldsymbol{p}, \boldsymbol{f}\, [\texttt{Time}(\boldsymbol{t}) \otimes \texttt{P}(\boldsymbol{p}, \texttt{new}) \otimes \boldsymbol{E}(\boldsymbol{p}, \boldsymbol{f}) \otimes \texttt{F}(\boldsymbol{f}, \texttt{ready})].$$



9 A Newcomer Takes a Seat: Act 2

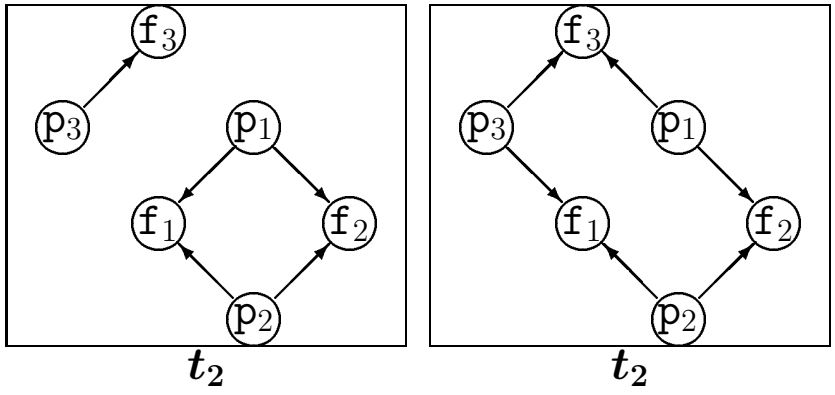
The event:

> **"A newcomer p (with his own fork f) takes a seat by wedging himself between a philosopher p_1 and a fork f_1 allocated to p_1 beforehand, and putting fork f down on the table,"**

is axiomatized as:

Axiom SEAT:

$$[\mathtt{Time}(\boldsymbol{t}) \otimes \boxed{\mathtt{P}(\boldsymbol{p}, \mathtt{new})} \otimes \boldsymbol{E}(\boldsymbol{p}, \boldsymbol{f}) \otimes \mathtt{P}(\boldsymbol{p_1}, \mathtt{idle}) \otimes \boxed{\boldsymbol{E}(\boldsymbol{p_1}, \boldsymbol{f_1})}\,]$$
$$\vdash$$
$$[\mathtt{Time}(\boldsymbol{t}) \otimes \boxed{\mathtt{P}(\boldsymbol{p}, \mathtt{idle})} \otimes \boldsymbol{E}(\boldsymbol{p}, \boldsymbol{f}) \otimes \mathtt{P}(\boldsymbol{p_1}, \mathtt{idle}) \otimes \boxed{\boldsymbol{E}(\boldsymbol{p}, \boldsymbol{f_1}) \otimes \boldsymbol{E}(\boldsymbol{p_1}, \boldsymbol{f})}\,].$$



10 A Philosopher Starts (Stops) Eating

The event

> **"Philosopher p needs to have two adjacent forks f_1 and f_2 to start eating,"**

is axiomatized as:

Axiom EAT:

$$[\mathtt{Time}(\boldsymbol{t}) \otimes \boxed{\mathtt{P}(\boldsymbol{p}, \mathtt{idle})} \otimes \boldsymbol{E}(\boldsymbol{p}, \boldsymbol{f_1}) \otimes \boldsymbol{E}(\boldsymbol{p}, \boldsymbol{f_2}) \otimes \boxed{\mathtt{F}(\boldsymbol{f_1}, \mathtt{ready}) \otimes \mathtt{F}(\boldsymbol{f_2}, \mathtt{ready})}\,]$$
$$\vdash$$
$$[\mathtt{Time}(\boldsymbol{t}) \otimes \boxed{\mathtt{P}(\boldsymbol{p}, \mathtt{eat})} \otimes \boldsymbol{E}(\boldsymbol{p}, \boldsymbol{f_1}) \otimes \boldsymbol{E}(\boldsymbol{p}, \boldsymbol{f_2}) \otimes \boxed{\mathtt{F}(\boldsymbol{f_1}, \mathtt{busy}) \otimes \mathtt{F}(\boldsymbol{f_2}, \mathtt{busy})}\,]$$

The event

> **"Philosopher p stops eating, which would release its two adjacent forks f_1 and f_2,"**

is axiomatized as:

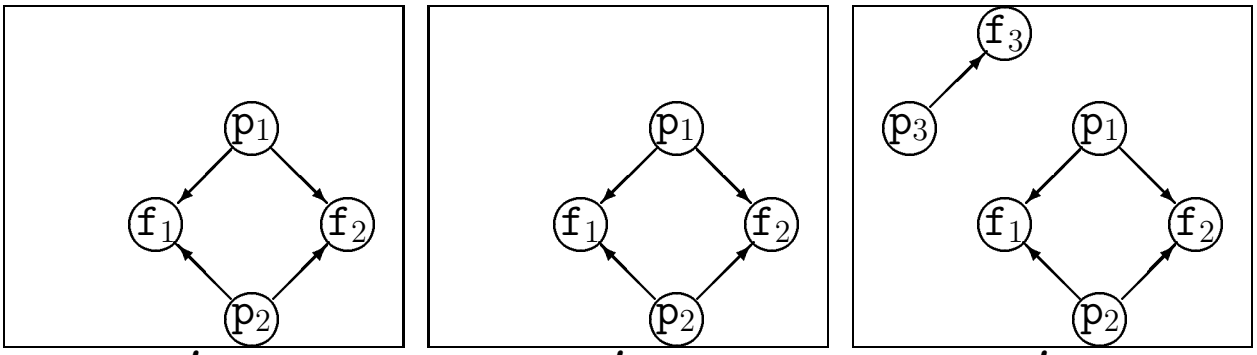
Axiom IDLE:

$$[\mathtt{Time}(\boldsymbol{t}) \otimes \boxed{\mathtt{P}(\boldsymbol{p}, \mathtt{eat})} \otimes \boldsymbol{E}(\boldsymbol{p}, \boldsymbol{f_1}) \otimes \boldsymbol{E}(\boldsymbol{p}, \boldsymbol{f_2}) \otimes \boxed{\mathtt{F}(\boldsymbol{f_1}, \mathtt{busy}) \otimes \mathtt{F}(\boldsymbol{f_2}, \mathtt{busy})}\,]$$
$$\vdash$$
$$[\mathtt{Time}(\boldsymbol{t}) \otimes \boxed{\mathtt{P}(\boldsymbol{p}, \mathtt{idle})} \otimes \boldsymbol{E}(\boldsymbol{p}, \boldsymbol{f_1}) \otimes \boldsymbol{E}(\boldsymbol{p}, \boldsymbol{f_2}) \otimes \boxed{\mathtt{F}(\boldsymbol{f_1}, \mathtt{ready}) \otimes \mathtt{F}(\boldsymbol{f_2}, \mathtt{ready})}\,]$$

11 Time is Ticking

The time advance is specified by:
Axiom TICK:

$$\texttt{Time}(t) \;\vdash\; \texttt{Time}(t+\varepsilon)$$

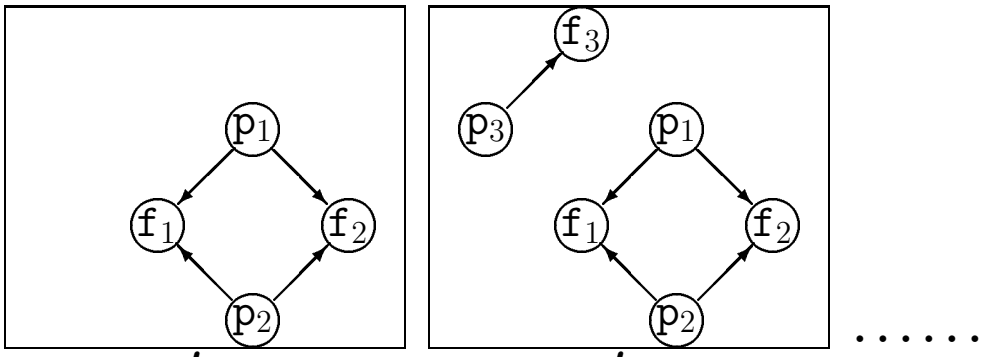


As "axioms of reals" we take all true elementary facts:

$$\begin{aligned}
&\vdash (1 < 2),\\
&\vdash (\sqrt{2}+\sqrt{5} < \sqrt{3}+2),\\
&\vdash (1 < +\infty),\\
&\ldots
\end{aligned}$$

12 The Adequacy Problem

(A) Scenarios (Trajectories) $\Longrightarrow$ Derivations !!!



t_0 t_1

(B) Derivations $\Longrightarrow$ Scenarios ???
"too many misleading derivations":

E. g., $$\frac{\texttt{Time}(t) \vdash \texttt{Time}(t+\varepsilon)}{\texttt{Time}(t) \vdash (\texttt{Time}(t) \otimes \texttt{Time}(t+\varepsilon))}$$

BUT not in LL.

Theorem 12.1 (Kanovich, LICS'92)
The Horn fragment of linear logic is complete w.r.t. the trajectory semantics.

Scenarios leading from $\texttt{Init}(t_0)$ ***to*** $\texttt{Goal}(t)$

$\Downarrow \Uparrow$

Linear logic derivations for: $\Gamma, W(t_0) \vdash Z(t)$

13 Quantitative Constraints vs Markov Property

A *lower-bound constraint*, **or** *delay*:

> **"It takes at least E min. to finish a piece of the fish after philosopher p starts eating."**

stipulates that:

(i) **when p started to `eat` at some moment t_1, and stopped to `eat` at a moment t_2 afterwards (if any), the following should hold:**

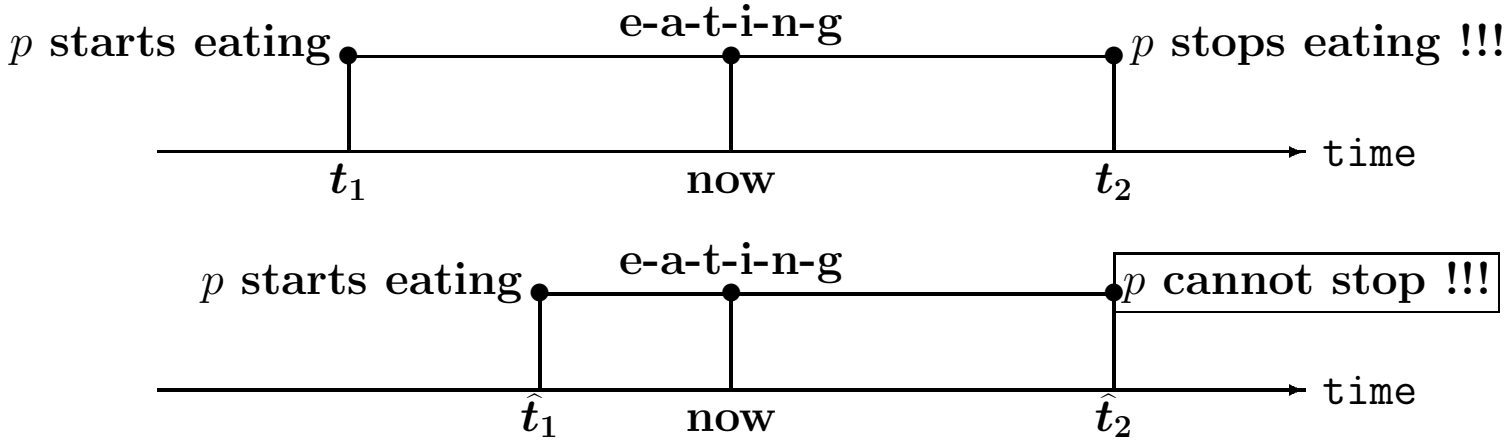
$$t_2 \geq t_1 + E,$$

(ii) **whereas, starting to `eat` at $\hat{t}_1$, our philosopher cannot stop `eating` at whatever moment $\hat{t}_2$ such that:**

$$\hat{t}_2 < \hat{t}_1 + E,$$

(a) **The Present determines the possible Future.**
Markov processes, the Cauchy problem.

(b) **The Future depends on the Past.**
Differential equations with delayed arguments.



14 Obstructions: Proofs are Always Markovian

$$\frac{\boldsymbol{\pi}_{\texttt{Past,Now}} : \texttt{Past} \vdash \texttt{Now} \qquad \boldsymbol{\pi}_{\texttt{Now,Future}} : \texttt{Now} \vdash \texttt{Future}}{\boldsymbol{\gamma}(\boldsymbol{\pi}_{\texttt{Past,Now}}, \boldsymbol{\pi}_{\texttt{Now,Future}}) : \texttt{Past} \vdash \texttt{Future}}$$

Corollary 14.1
Suppose that π_2 proves a sequent of the form:

$$C(t, s) \vdash C(t_2, s_2).$$

Then whatever proof π' of a sequent of the form:

$$C(t_1, s_1) \vdash C(t, s),$$

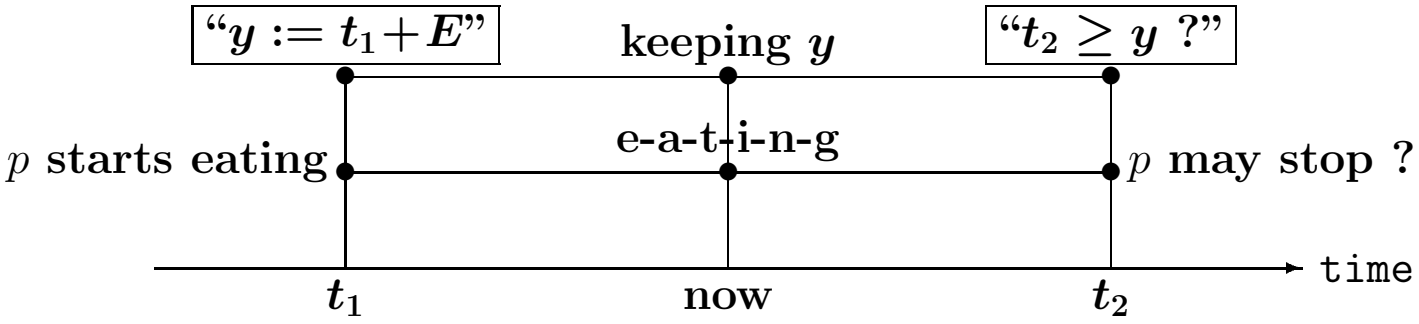
we take, $\gamma(\pi', \pi_2)$ is always a legal proof of the sequent:

$$C(t_1, s_1) \vdash C(t_2, s_2).$$

15 "Time-keepers" to keep the Markov property

"It takes at least E min. to finish a piece of the fish"

$L(p, y)$:= "y is a number recorded by a secretary of p".



Axiom IDLE (adjusted with $\boxed{\boxed{L(p,y)}}$):

$$[\mathtt{Time}(t) \otimes \boxed{\mathtt{P}(p, \mathtt{eat})} \otimes E(p, f_1) \otimes E(p, f_2) \otimes \boxed{\mathtt{F}(f_1, \mathtt{busy}) \otimes \mathtt{F}(f_2, \mathtt{busy})} \otimes \boxed{\boxed{L(p,y) \otimes (t \geq y)}}\,] \vdash [\mathtt{Time}(t) \otimes \boxed{\mathtt{P}(p, \mathtt{idle})} \otimes E(p, f_1) \otimes E(p, f_2) \otimes \boxed{\mathtt{F}(f_1, \mathtt{ready}) \otimes \mathtt{F}(f_2, \mathtt{ready})} \otimes \boxed{\boxed{L(p, -\infty)}}\,]$$

Axiom EAT (adjusted with $\boxed{\boxed{L(p,y)}}$):

$$[\mathtt{Time}(t) \otimes \boxed{\mathtt{P}(p, \mathtt{idle})} \otimes E(p, f_1) \otimes E(p, f_2) \otimes \boxed{\mathtt{F}(f_1, \mathtt{ready}) \otimes \mathtt{F}(f_2, \mathtt{ready})} \otimes \boxed{\boxed{L(p,y)}}\,] \vdash [\mathtt{Time}(t) \otimes \boxed{\mathtt{P}(p, \mathtt{eat})} \otimes E(p, f_1) \otimes E(p, f_2) \otimes \boxed{\mathtt{F}(f_1, \mathtt{busy}) \otimes \mathtt{F}(f_2, \mathtt{busy})} \otimes \boxed{\boxed{L(p, t+E)}}\,]$$

16 Four Final Horn Axioms: $A(\overline{x},\overline{a}) \vdash \exists \overline{y}\, B(\overline{x},\overline{y},\overline{b})$

Axiom NEW:

$$\mathtt{Time}(t) \vdash \exists p, f\, [\mathtt{Time}(t) \otimes \mathtt{P}(p,\mathtt{new}) \otimes E(p,f) \otimes \mathtt{F}(f,\mathtt{ready})].$$

Axiom SEAT:

$$[\mathtt{Time}(t) \otimes \boxed{\mathtt{P}(p,\mathtt{new})} \otimes E(p,f) \otimes \mathtt{P}(p_1,\mathtt{idle}) \otimes \boxed{E(p_1,f_1)}\,]$$
$$\vdash$$
$$[\mathtt{Time}(t) \otimes \boxed{\mathtt{P}(p,\mathtt{idle})} \otimes E(p,f) \otimes \mathtt{P}(p_1,\mathtt{idle}) \otimes \boxed{E(p,f_1) \otimes E(p_1,f)}\,].$$

Axiom EAT:

$$[\mathtt{Time}(t) \otimes \boxed{\mathtt{P}(p,\mathtt{idle})} \otimes E(p,f_1) \otimes E(p,f_2) \otimes \boxed{\mathtt{F}(f_1,\mathtt{ready}) \otimes \mathtt{F}(f_2,\mathtt{ready})} \otimes \boxed{L(p,y)}\,]$$
$$\vdash$$
$$[\mathtt{Time}(t) \otimes \boxed{\mathtt{P}(p,\mathtt{eat})} \otimes E(p,f_1) \otimes E(p,f_2) \otimes \boxed{\mathtt{F}(f_1,\mathtt{busy}) \otimes \mathtt{F}(f_2,\mathtt{busy})} \otimes \boxed{L(p,t+E)}\,]$$

Axiom IDLE:

$$[\mathtt{Time}(t) \otimes \boxed{\mathtt{P}(p,\mathtt{eat})} \otimes E(p,f_1) \otimes E(p,f_2) \otimes \boxed{\mathtt{F}(f_1,\mathtt{busy}) \otimes \mathtt{F}(f_2,\mathtt{busy})} \otimes \boxed{L(p,y) \otimes (t \geq y)}\,]$$
$$\vdash$$
$$[\mathtt{Time}(t) \otimes \boxed{\mathtt{P}(p,\mathtt{idle})} \otimes E(p,f_1) \otimes E(p,f_2) \otimes \boxed{\mathtt{F}(f_1,\mathtt{ready}) \otimes \mathtt{F}(f_2,\mathtt{ready})} \otimes \boxed{L(p,-\infty)}\,]$$

Axiom TICK: **(should respect all upper bounds)**

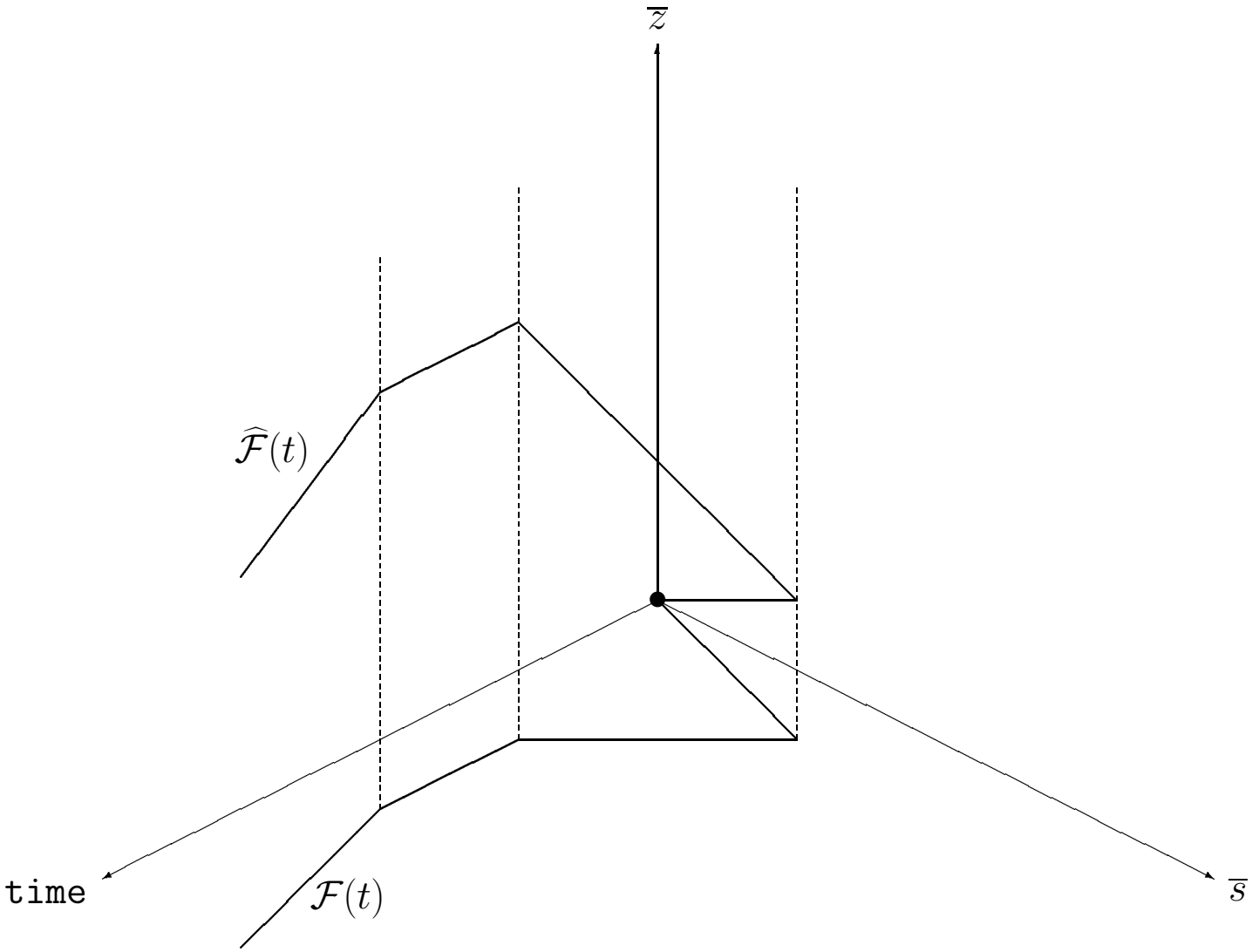
$$\mathtt{Time}(t) \vdash \mathtt{Time}(t+\varepsilon)$$

17 Trajectories as Projections: Hidden Z

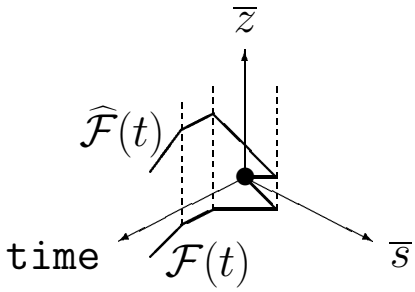
"Markovian" trajectories $\widehat{\mathcal{F}}$: TIME $\longrightarrow$ STATE $\times$ Z.

projection

"Non-Markovian" trajectories $\mathcal{F}$: TIME $\longrightarrow$ STATE.



18 The Adequacy Problem



Trajectories $\widehat{\mathcal{F}}$ such that: $\widehat{\mathcal{F}}(t) = (\overline{s}, \overline{z})$.

$\Downarrow \Uparrow$

LL derivations for: $C(t_0, \overline{s}_0, \overline{z}_0) \vdash_T C(t, \overline{s}, \overline{z})$.

Trajectories $\mathcal{F}$ such that: $\mathcal{F}(t) = \overline{s}$.

$\Downarrow \Uparrow$

LL derivations for: $C(t_0, \overline{s}_0, \overline{z}_0) \vdash_T \exists \overline{z}\, C(t, \overline{s}, \overline{z})$.

Trajectories $\mathcal{F}$ such that, for some t: $\mathcal{F}(t) = \overline{s}$.

$\Downarrow \Uparrow$

LL derivations for: $C(t_0, \overline{s}_0, \overline{z}_0) \vdash_T \exists t \exists \overline{z}\, C(t, \overline{s}, \overline{z})$.

19 Coarse Hour-Glass + Precise One-Hand Watch

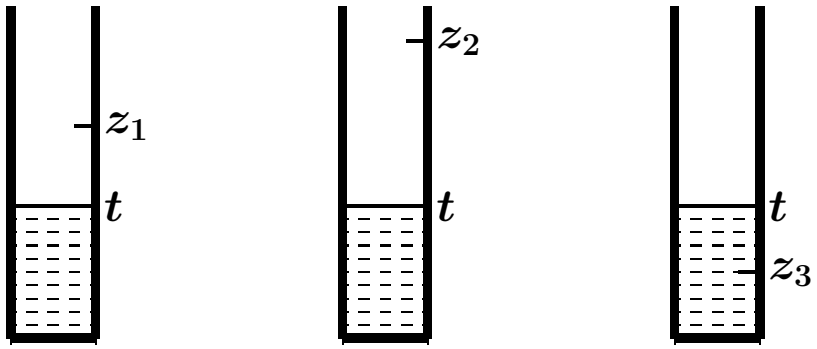
The bisimulation equivalence between "Markovian" trajectories $\widehat{\mathcal{F}} : \mathtt{TIME} \longrightarrow \mathtt{STATE} \times \mathtt{Z}$, is based on invariance of the real-time systems under the translation

$$\mathtt{time} \longmapsto \mathtt{time} + \alpha.$$

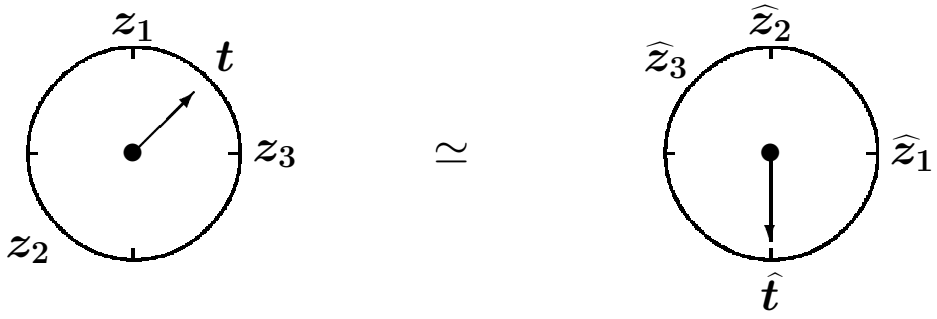
Invariants: Discrete part + Continuous part:

(a) The coarse relative time intervals between "now" and each of the registered numbers.

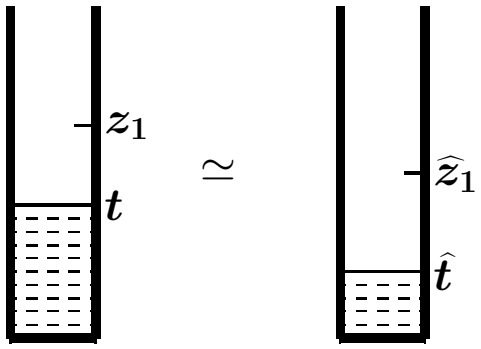
E. g., "$1 < z_1 - t \leq 2$", "$2 \leq z_2 - t < 3$", "$z_3 - t < 0$".



(b) The absolute time distribution of all the registered numbers together but on the "circular" one-hand watch:



20 The Individual Invariants on the Hour-Glass



whenever: $z_1 - t \approx \widehat{z}_1 - \hat{t}$. E. g., $1 < z_1 - t,\ \widehat{z}_1 - \hat{t} \leq 2$.

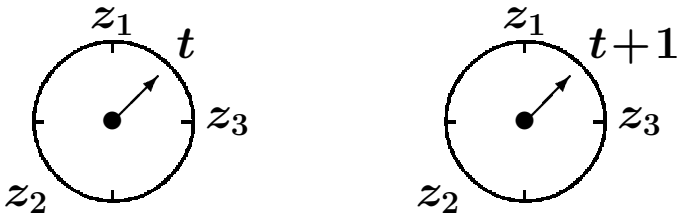
Lemma 20.1 ("An Action")
Let $C(t,\ \overline{s},\ z_1, z_2, z_3, ...) \simeq C(\hat{t},\ \overline{s},\ \widehat{z}_1, \widehat{z}_2, \widehat{z}_3, ...)$.
Suppose that an action β ***is able to transform***
$C(t,\ \overline{s},\ z_1, z_2, z_3, ...)$ ***into*** $C(t,\ \overline{s}',\ z'_1, z'_2, z'_3, ...)$.
Then the action β ***is also able to transform***
$C(\hat{t},\ \overline{s},\ \widehat{z}_1, \widehat{z}_2, \widehat{z}_3, ...)$ ***into*** $C(\hat{t},\ \overline{s}',\ \widehat{z}'_1, \widehat{z}'_2, \widehat{z}'_3, ...)$,
and, besides,
$C(t,\ \overline{s}',\ z'_1, z'_2, z'_3, ...) \simeq C(\hat{t},\ \overline{s}',\ \widehat{z}'_1, \widehat{z}'_2, \widehat{z}'_3, ...)$.

Lemma 20.2 ("A Long Tick")
Let $C(t,\ \overline{s},\ z_1, z_2, z_3, ...) \simeq C(\hat{t},\ \overline{s},\ \widehat{z}_1, \widehat{z}_2, \widehat{z}_3, ...)$.
Then
$C(t+1,\ \overline{s},\ z_1, z_2, z_3, ...) \simeq C(\hat{t}+1,\ \overline{s},\ \widehat{z}_1, \widehat{z}_2, \widehat{z}_3, ...)$.

Proof.

(a) If $z_1 - t \approx \widehat{z}_1 - \hat{t}$, then $z_1 - (t+1) \approx \widehat{z}_1 - (\hat{t}+1)$.

(b)

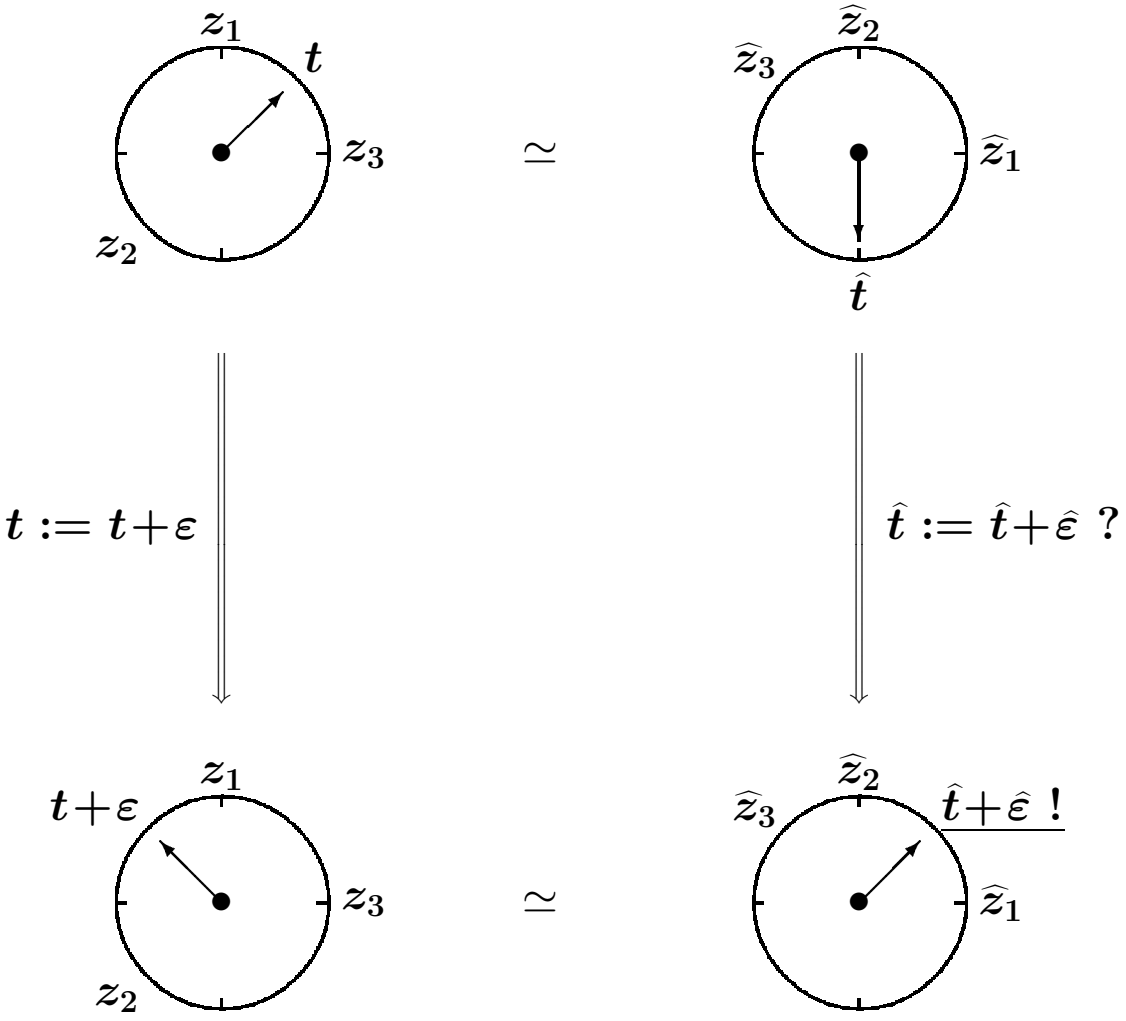


21 The All-Together Invariant on the Watch

Lemma 21.1 ("A Short Tick")
Let $C(t,\ \overline{s},\ z_1, z_2, z_3, ...) \ \simeq\ C(\hat{t},\ \overline{s},\ \widehat{z}_1, \widehat{z}_2, \widehat{z}_3, ...)$.

Then for any ε, ***one can find an*** $\hat{\varepsilon}$ ***such that*** $C(t{+}\varepsilon,\ \overline{s},\ z_1, z_2, z_3, ...) \ \simeq\ C(\hat{t}{+}\hat{\varepsilon},\ \overline{s},\ \widehat{z}_1, \widehat{z}_2, \widehat{z}_3, ...)$.

Proof. Each of these absolute $z_1, z_2, z_3, ...$ is immovable. A key point of the "continuous part":



22 Is Real Time Really Continuous ?

Lemma 22.1 ("A Short Tick")
Let $C(t, \overline{s}, z_1, z_2, z_3, ...) \simeq C(\hat{t}, \overline{s}, \widehat{z}_1, \widehat{z}_2, \widehat{z}_3, ...)$.
Then for any ε, ***one can find an*** $\hat{\varepsilon}$ ***such that***
$C(t+\varepsilon, \overline{s}, z_1, z_2, z_3, ...) \simeq C(\hat{t}+\hat{\varepsilon}, \overline{s}, \widehat{z}_1, \widehat{z}_2, \widehat{z}_3, ...)$,
and, besides, $\hat{\varepsilon}$ ***can be chosen as a simple rational combination of*** $\widehat{z}_1$, $\widehat{z}_2$, $\widehat{z}_3$, ...

Theorem 22.1 ***Let all bounds be commensurable. that is, exactly divisible by the same unit an integral number of times.***

Then any legal trajectory $\mathcal{F} : \texttt{TIME} \longrightarrow \texttt{STATE}$ ***can be adjusted to a legal trajectory*** $\widetilde{\mathcal{F}} : \texttt{TIME} \longrightarrow \texttt{STATE}$ ***so that***

(a) $\widetilde{\mathcal{F}}$ ***matches just the same scenarios associated with*** $\mathcal{F}$, ***and***

(b) ***each of the discontinuity points of*** $\widetilde{\mathcal{F}}$ ***is rational.***

Corollary 22.1 ***If all bounds are commensurable, then:***

Trajectory semantics with 'Continuous Time'

⇓ ⇑

Trajectory semantics with 'Rational Time'

23 Concluding Remarks

(a) The comprehensive logical system that automatically exploits peculiarities of the systems in question.

(b) Spec:
"Naïve", Flexible, Easy-to-specify, Easy-to-modify. Easy-to-catch-a-mistake.

(b1) Instant Events $\Longleftrightarrow$
Horn Axioms: $\texttt{Pre}(t, \overline{s}) \vdash \texttt{Post}(t, \overline{s}')$.

(b2) Time advance $\Longleftrightarrow$
Horn Axioms: $\texttt{Time}(t) \vdash \texttt{Time}(t+\varepsilon)$.

(b3) Time Constraints $\Longleftrightarrow$
Horn Axioms: $\texttt{Pre}(t, \overline{s}, \overline{z}) \vdash \texttt{Post}(t, \overline{s}', \overline{z}')$.

(c) Comprehensive = "Derivations $\Longleftrightarrow$ Scenarios"

(d) Exec:
Complexity in accordance with the original systems.

(e) Higher-Order Problems:

(e1) reachability, safety, liveness, deadlock, schedulability, membership with incomplete information,

(e2) protocol analysis, simulation, monitoring, diagnosis,

(e3) stability of topology, connectedness, acyclicity, etc.