ASSUMED KNOWLEDGE

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The course is based on the calculus course studied in High School. Since the course is rather difficult, it is very important not to overestimate you knowledge before the course. If you feel not ready, it is better to start with 103 calculus course. To make it easier to you to decide whether your are well prepared, I give a short sketch of required notions and problems to test yourself. If you are doing well, then the problems will be easy for you. Do not write down full solutions, only make sure that you know how to solve the problems.

1. A function $f : A \to B$ is a rule which associates to each element x in a set A an element y in a set B. We will be interested in the case when A and B are subsets of the set $\mathbf{R}$ of real numbers. We call A the domain of f and call B the range of f.

Test problems 1.1/5-9 (section 1.1, exercises 5-9).

2. Actions on functions: $h = f + g$, $h = fg$, $h = f^n$, etc., are defined by requiring that $h(x) = f(x) + g(x)$, etc. The composition $h = f \circ g$ is defined by $h(x) = f(g(x))$. (What about the domains?)

Test problems 1.3/9,11,13,31,45-50.

3. Limits $\lim_{x\to a} f(x)$, $\lim_{x\to a^+} f(x)$, $\lim_{x\to -\infty} f(x)$, etc. Basic properties of limits: $\lim_{x\to a} f(x)g(x) = \lim_{x\to a} f(x) \cdot \lim_{x\to a} g(x)$, etc.

Test problems 2.3/2,20-23,45.

4. Continuity: f is continuous at a if $f(a) = \lim_{x\to a} f(x)$. Basic properties: sum, product, composition, etc., of continuous functions are continuous. Intermediate Value Theorem.

Test problems 2.5/8,9,32.

5. Derivative is defined as $f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$. If $y = f(x)$, then we denote also $\frac{dy}{dx} = f'(x)$. Interpretation as the slope of the tangent. Connection to velocity. Basic formulas: $(fg)' = fg' + f'g$, etc. The chain rule $(f \circ g)'(x) = f'(g(x))g'(x)$.

Test problems 2.6.15, 3.3/22, 3.5/10, 3.6/29-32.

6. Connection between the derivative (and the second derivative) and the graph of a function: increasing, decreasing, local maximum and minimum, concaveness upward and downward, inflection point.

Test problems 4.1/45-46, 4.3/3, 4.5/25,39.

7. Differential $dy = f'(x)dx$, where $y = f(x)$. Linearization of $f(x)$ around x: the approximation $f(x + \Delta x) = f(x) + \Delta y \approx f(x) + f'(x)\Delta x$.

Test problems 3.10/31-36,45.

8. Antiderivative $F(x) = \int f(x)dx$ of $f(x)$ is defined (not uniquely) by requiring that $F'(x) = f(x)$.

Test problems 4.10/1,12,14.

9. Area, Riemann sums, definite integrals $\int_a^b f(x)dx$ and the fundamental theorem of calculus: $g(x) = \int_a^x f(t)dt$ is an antiderivative of $f(x)$.

Test problems 5.3/20-30.

10. Integrating by substitution: $\int f(g(x))g'(x)dx = \int f(g(x))dg(x) = \int f(t)dt$, where $t = g(x)$.

Test problems 5.5/1-6,37-40.