

Algebra and Trigonometry HW 4

June 5, 2006

1. EXERCISE SET 4.1

1. For each of the following functions,

(a) find the vertex,

(b) find the line of symmetry, and

(c) determine whether there is a maximum or minimum function value and find that value.

(1) $f(x) = x^2$

(2) $f(x) = 2(x - 9)^2$

(3) $f(x) = 2(x - 1)^2 - 4$

2. For each of the following functions,

(a) use completing the square to put each equation into the form

$$f(x) = a(x - h)^2 + k,$$

(b) find the vertex, and

(c) determine whether there is a maximum or minimum function value and find that value.

(1) $f(x) = -x^2 + 2x + 3$

(2) $f(x) = x^2 + 3x$

(3) $f(x) = 3x^2 + x - 4$

3. Graph.

(1) $f(x) = x^2 - 3x - 4$

(2) $f(x) = 2x^2 - 4x - 2$

(3) $f(x) = x^2 - 8x + 19$

4. Find the x - intercepts.

(1) $f(x) = x^2 + 2x - 3$

(2) $f(x) = x^2 - 3x - 4$

(3) $f(x) = -5x^2 + 6x - 5$

5. Find a such that

$$f(x) = ax^2 + 3x - 8$$

has a minimum value at $x = -2$.

6. Find b such that

$$f(x) = -4x^2 + bx + 3$$

has a maximum value of 50.

7. Find c such that

$$f(x) = -0.2x^2 - 3x + c$$

has a maximum value of -225 .

2. EXERCISE SET 4.2

1. Find the union or the intersection.

(1) $\{3, 4, 5, 8, 10\} \cap \{1, 2, 3, 4, 5, 6, 7\}$

(2) $\{3, 4, 5, 8, 10\} \cup \{1, 2, 3, 4, 5, 6, 7\}$

(3) $\{0, 2, 4, 6, 8\} \cup \{4, 6, 9\}$

(4) $\{0, 2, 4, 6, 8\} \cap \{4, 6, 9\}$

(5) $\{a, b, c\} \cup \{c, d\}$

(6) $\{a, b, c\} \cap \{c, d\}$

2. Graph.

(1) $\{x|7 \leq x\} \cup \{x|x < 9\}$

(2) $\{x|-\frac{1}{2} \leq x\} \cap \{x|x < \frac{1}{2}\}$

(3) $\{x|x \geq 5\} \cup \{x|x \leq -3\}$

(4) $\{x|x \geq -3\} \cup \{x|x \leq 0\}$

3. Solve.

(1) $-2 \leq x + 1 < 4$

(2) $-3 \leq x + 4 \leq 3$

(3) $3x \leq -6$ or $x - 1 > 0$

(4) $-3 < 1 - 2x \leq 3$

3. EXERCISE SET 4.3

1. Solve and graph.

(1) $|x| = 7$

(2) $|x| \geq 4.5$

(3) $|x| > 7$

2. Solve. Use any method.

(1) $|x - 1| = 4$

(2) $|x + 8| < 9$

(3) $|x + 6| > 10$

(4) $|x| < -3$

(5) $|3x + 5| < 0$

(6) $|x + 2| \leq |x - 5|$

(7) $|x| - |x - 3| < 7$

(8) $|x - 3| + |2x + 5| > 6$

4. EXERCISE SET 4.4

1. Solve. Use interval notation for the answer, where appropriate.

$$(1) (x + 5)(x - 3) > 0$$

$$(2) (x - 1)(x + 2) \leq 0$$

$$(3) x^2 + x - 2 < 0$$

$$(4) x^2 + 8 < 6x$$

$$(5) (x - 1)(x + 2)(x - 4) < 0$$

$$(6) (x - 2)(x - 3)(x + 1) < 0$$

$$(7) \frac{1}{4-x} < 0$$

$$(8) \frac{-4}{2x+5} > 0$$

$$(9) \frac{3x+2}{x-3} > 0$$

$$(10) \frac{x+1}{x+2} \leq 3$$

$$(11) \frac{x}{x+3} > -1$$

$$(12) (x - 4)(x + 3) > (x - 1)^2$$

$$(13) x + \frac{4}{x} > 4$$

$$(14) \frac{4}{x^2} - 1 > 0$$

$$(15) \left| \frac{x+3}{x-4} \right| < 2$$

$$(16) |x^2 - 5| = 5 - x^2$$

$$(17) (1 - x)^3$$

$$(18) \left| 1 + \frac{1}{x} \right| < 3$$

$$(19) |x|^2 - 4|x| + 4 \geq 9$$

$$(20) \left| 2 - \frac{1}{x} \right| \leq 2 + \left| \frac{1}{x} \right|$$

5. EXERCISE SET 4.5

1. Sketch a graph of the polynomial function.

$$(1) f(x) = \frac{1}{3}x^6$$

$$(2) f(x) = (x + 3)(x - 2)(x + 1)$$

$$(3) f(x) = x^4 - x^3$$

$$(4) f(x) = x^3 - 3x^2 - 4x + 12$$

$$(5) f(x) = -3x^4 - x^3 + 2x^2$$

$$(6) f(x) = x(x - 1)(x - 2)(x - 3)$$

6. EXERCISE SET 4.6

1. Determine whether 2, 3, and -1 are roots, or zeros, of

$$P(x) = 2x^3 - 3x^2 + x - 1.$$

2. Use synthetic division to find the quotient and the remainder.

(1) $(2x^4 + 7x^3 + x - 12) \div (x + 3)$

(2) $(x^3 - 7x^2 + 13x + 3) \div (x - 2)$

(3) $(x^5 + 32) \div (x + 2)$

3. Factor the polynomial $P(x)$. Then solve the equation $P(x) = 0$.

(1) $P(x) = x^3 + 4x^2 + x - 6$

(2) $P(x) = x^3 - 6x^2 + 3x + 10$

(3) $P(x) = x^3 - 3x^2 - 10x + 24$

(4) $P(x) = x^4 - x^3 - 19x^2 + 49x - 30$

7. EXERCISE SET 4.7

1. Find the roots of the polynomial or polynomial equation, and state the multiplicity of each.

(1) $P(x) = (x + 3)^2(x - 1)$

(2) $P(x) = x^4 - 4x^2 + 3$

(3) $P(x) = x^3 + 3x^2 - x - 3$

(4) $P(x) = x^3 - x^2 - 2x + 2$

2. Find a polynomial of degree 3 with the given numbers as roots.

(1) 2, 3, 5

(2) 2, i , $-i$

(3) -3 , $2i$, $-2i$

3. Find the rational roots, if they exist, of the polynomial or equation. If possible, find the other roots. Then, write the equation or polynomial in factored form.

(1) $x^3 + 3x^2 - 2x - 6$

(2) $x^3 - 3x + 4$

(3) $3x^4 - 4x^3 + x^2 + 6x - 2$

(4) $x^3 - 8x^2 + 17x - 4$

4. Solve the equation. Use synthetic division, the quadratic formula, the theorems of this section, or whatever else you think might help.

(1) $x^3 - 4x^2 + x - 4 = 0$

(2) $x^3 - x^2 - 7x + 15 = 0$

(3) $x^4 + 9x^2 - 112 = 0$