

NAME: _____

*Partial credit may be given for substantial work. Little or no credit will be given for answers with no supporting work shown, even if you write (guess?) the correct answer, but you do not need to show work in questions labelled **NPC** (no partial credit). If you change an answer, please either erase or cross out the answer you do not want considered; questions with more than one answer will be marked wrong. Credit will not be given for simply quoting a formula. Please use a pen to write your exam answers. Each question is worth 2 marks: you get full credit for perfect work, 1 mark for work that is almost perfect, and 0 for non-substantial work. Credit will not be given for merely quoting a formula.*

1. Find the equation of the tangent line to the curve $y = x^3 + x^{1/3}$ at $x = 1$.
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2. Find $f'(\frac{\pi}{3})$ if $f(x) = \frac{x}{\sin x + \cos x}$.
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Situation: Consider a function $f : \mathbb{R} \rightarrow \mathbb{R}$ whose first and second derivatives f' and f'' exists everywhere. Problems 3 through 4 refer to this situation.

3. If $a(x) = x^2 f(x^2)$, find $a''(x)$ in terms of f , f' and f'' .

4. If $b(x) = \frac{x + f(x)}{x - f(x)}$, find $b'(x)$ in terms of f and f' .

Situation: Let f be an everywhere differentiable function and let $g = f \circ f$. Problems 5 through 6 refer to this situation.

5. Find $g'(x^2)$ by using f and f' .

6. Find $g''(x^2)$ by using f , f' and f'' .

Situation: Let f be a function defined for all real numbers, which satisfies

$$f(x+h) - f(x) = (x^2 + h)^2 - (x^2 - h)^2.$$

Problems 6a through 6c refer to this situation. (Hint: Expand the squares and simplify first!)

(a) Find $f'(x)$.

(b) Find $f''(x)$.

(c) Find $f'''(x)$.

7. A ball is tossed upwards and its height above the ground is given by $h(t) = 48t - 16t^2$. At the time $t = 1$, what is its instantaneous velocity?
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8. A cylindrical swimming pool is being filled from a fire hose at rate of 5 cubic feet per second. If the pool is 40 feet across, how fast is the water level increasing when the pool is one third full?
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9. Find the slopes of the two tangent lines to the parabola $y = x^2 + 2x$ passing through the point $(-4, -4)$.

10. Find $f'(x)$ if it is known that $\frac{d}{dx} \left(f \left(\frac{x-1}{x+1} \right) \right) = 1 - 2x$.

11. Find y' in terms of y and x given that

$$\tan y + y \sin x = 3.$$

12. Let a and b be non-zero constants. If $f(x) = \frac{x^2 - ax}{x^2 - bx}$, find $f'(ab)$.

13. A little boy buys a spherical balloon of total volume 1 cubic foot. He starts blowing to fill the balloon at a rate of 0.1 cubic feet per minute. How fast is the radius of the balloon increasing when he has the balloon halfway blown up?
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14. Find $f'(\pi)$ if $f(x) = \frac{x \cos x}{x + 1}$.
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15. Find the equation of the tangent line to the graph of the equation $x^2 + 4y^2 = 16$ at the point $(-2, -\sqrt{3})$.

16. Suppose the position of an accelerating bobsled is given by $s(t) = 5t^{3/2}$ feet .

(a) What is the speed of the bobsled when $t = 1$?

(b) What is the acceleration when $t = 1$?
