Quiz 1 solution for Math104, 2008

1. Find the area of the region bounded by the given curves.

$$y = xe^{x^2}, \quad y = 0, \quad x = 3$$

Ans. Let $u = x^2$. Then $du = 2xdx$.

$$\begin{aligned} \text{Area} = \int_0^3 xe^{x^2} dx &= \frac{1}{2}\int_0^9 e^u du \\ &= \frac{1}{2}\left[e^u\right]_0^9 \\ &= \frac{1}{2}\left(e^9 - 1\right). \end{aligned}$$

2. Find the volume obtained by rotating the region bounded by the given curves about the specified axis.

$$x = 0, \quad x = 9 - y^2, \quad \text{about } x = -1$$

Ans.

$$\begin{aligned} \text{V} &= \pi\int_{-3}^3 \{[(9-y^2)-(-1)]^2 - [0-(-1)]^2\} dy \\ &= 2\pi\int_0^3 [(10-y^2)^2 - 1] dy \\ &= 2\pi\int_0^3 (99 - 20y^2 + y^4) dy \\ &= 2\pi\left[99y - \frac{20}{3}y^3 + \frac{1}{5}y^5\right]_0^3 = 2\pi\left(297 - 180 + \frac{243}{5}\right) \\ &= \frac{1656\pi}{5}. \end{aligned}$$

3. Differentiate the function.

$$y = e^{x\sin 2x}$$

Ans.

$$\begin{aligned} y' &= (x\sin 2x)' \cdot e^{x\sin 2x} \\ &= (\sin 2x + 2x\cos 2x) \cdot e^{x\sin 2x}. \end{aligned}$$