Name : ____

Quiz 2 Solution for Math104, 2008

1. Evaluate the integral.

$$\int \tan x \ln(\cos x) dx$$

Ans. Let $u = \ln(\cos x)$. Then $du = \frac{-\sin x}{\cos x} dx = -\tan x dx$.

$$\int \tan x \ln(\cos x) dx = \int -u du = -\frac{1}{2}u^2 + C$$

$$= -\frac{1}{2}[\ln(\cos x)]^2 + C.$$

2. Find the limit.

$$\lim_{x \to 1} \left(\frac{1}{\ln x} - \frac{1}{x-1} \right)$$

Ans. Make the common denominator :

$$\lim_{x \to 1} \left(\frac{1}{\ln x} - \frac{1}{x-1} \right) = \lim_{x \to 1} \frac{x - 1 - \ln x}{(x-1)\ln x} = \lim_{x \to 1} \frac{1 - \frac{1}{x}}{\frac{x-1}{x} + \ln x} \cdot \frac{x}{x}$$

$$= \lim_{x \to 1} \frac{x-1}{x - 1 + x \ln x} = \lim_{x \to 1} \frac{1}{1 + \ln x + 1}$$

$$= \frac{1}{2}$$

3. Evaluate the integral.

$$\int_0^2 x^3\sqrt{x^2+4}dx$$

Ans. Let $u = x^2+4$. Then $x^2 = u-4, \quad du = 2xdx$.

$$\begin{aligned}
\int_0^2 x^3\sqrt{x^2+4}dx &= \frac{1}{2}\int_4^8 (u-4)\sqrt{u}du \\
&= \frac{1}{2}\int_4^8 (u\sqrt{u}-4\sqrt{u})du \\
&= \frac{1}{2}\left[\frac{2}{5}u^2\sqrt{u}-\frac{8}{3}u\sqrt{u}\right]_4^8 \\
&= \frac{1}{2}\left(\frac{256}{5}\sqrt{2}-\frac{128}{3}\sqrt{2}-\frac{64}{5}+\frac{64}{3}\right) \\
&= \frac{64}{15}(\sqrt{2}+1)
\end{aligned}$$

Or, you can use the trigonometric substitution :
Let $x = 2\tan\theta, \quad dx = 2\sec^2\theta d\theta$.

$$\begin{aligned}
\int_0^2 x^3\sqrt{x^2+4}dx &= \int_0^{\frac{\pi}{4}} 8\tan^3\theta\cdot 2\sec\theta\cdot 2\sec^2\theta d\theta \\
&= 32\int_0^{\frac{\pi}{4}} \tan^2\theta\sec^2\theta\cdot\sec\theta\tan\theta d\theta \\
&= 32\int_1^{\sqrt{2}} (u^2-1)u^2du, \quad \text{(by letting } u=\sec\theta\text{)} \\
&= 32\int_1^{\sqrt{2}} (u^4-u^2)du \\
&= 32\left[\frac{1}{5}u^5-\frac{1}{3}u^3\right]_1^{\sqrt{2}} \\
&= 32\left[\left(\frac{4\sqrt{2}}{5}-\frac{2\sqrt{2}}{3}\right)-\left(\frac{1}{5}-\frac{1}{3}\right)\right] \\
&= 32\left(\frac{2}{15}\sqrt{2}+\frac{2}{15}\right) \\
&= \frac{64}{15}(\sqrt{2}+1)
\end{aligned}$$