

Quiz 3 for Math114, 2009 - 207, 208

1. Find the volume of the parallelepiped with adjacent edges PQ , PR and PS where

$$P(3, 0, 1), \quad Q(-1, 2, 5), \quad R(5, 1, -1), \quad S(0, 4, 2)$$

Ans.

$$\begin{aligned}\overrightarrow{PQ} &= \langle -4, 2, 4 \rangle \\ \overrightarrow{PR} &= \langle 2, 1, -2 \rangle \\ \overrightarrow{PS} &= \langle -3, 4, 1 \rangle\end{aligned}$$

So,

$$\overrightarrow{PR} \times \overrightarrow{PS} = \begin{vmatrix} i & j & k \\ 2 & 1 & -2 \\ -3 & 4 & 1 \end{vmatrix} = 9i + 4j + 11k.$$

$$\overrightarrow{PQ} \bullet (\overrightarrow{PR} \times \overrightarrow{PS}) = \langle -4, 2, 4 \rangle \bullet \langle 9, 4, 11 \rangle = 16.$$

Thus,

$$V = |\overrightarrow{PQ} \bullet (\overrightarrow{PR} \times \overrightarrow{PS})| = 16.$$

2. Find the equation for the line through the points $(6, 1, -3)$ and $(2, 4, 5)$.

Ans. The direction vector of the line is

$$\vec{d} = \langle 4, -3, -8 \rangle$$

So, the line equation is

$$\frac{x-6}{4} = \frac{y-1}{-3} = \frac{z+3}{-8}$$

or

$$\begin{aligned}x &= 6 + 4t, \\ y &= 1 - 3t \\ z &= -3 - 8t\end{aligned}$$

Note that you can use the other point $(2, 4, 5)$ to get the line equation or direction vector $\vec{d} = \langle -4, 3, 8 \rangle$, and they all represent the same line.