Quiz 4 Solution for Math104, 2008

1. Find the length of the curve

$$x = 3t^2, \quad y = 2t^3, \quad 0 \le t \le 2.$$

Ans.

$$\begin{aligned}
L &= \int_0^2 \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt \\
&= \int_0^2 \sqrt{(6t)^2 + (6t^2)^2} dt = \int_0^2 \sqrt{36t^2 + 36t^4} dt \\
&= \int_0^2 \sqrt{36t^2}\sqrt{1+t^2} dt = \int_0^2 6t\sqrt{1+t^2} dt \\
&= 6\int_1^5 u^{1/2} \cdot \frac{1}{2} du \quad (u = 1 + t^2, du = 2tdt) \\
&= 3 \cdot \frac{2}{3}\left[u^{3/2}\right]_1^5 = 2\left(5^{3/2} - 1\right) \\
&= 2(5\sqrt{5} - 1)
\end{aligned}$$

2. Find the points of intersection of the curves $r = 2$ and $r = 4\cos\theta$.

Ans.

$$2 = 4\cos\theta \Rightarrow \cos\theta = \frac{1}{2}$$

$$\theta = \pm\frac{\pi}{3}$$

Thus, the intersection points are

$$\left(2, \frac{\pi}{3}\right), \quad \left(2, -\frac{\pi}{3}\right)$$

in polar coordinates, or $(1, \sqrt{3})$, $(1, -\sqrt{3})$ in Cartesian coordinates.
(Note that $r = 4\cos\theta \Rightarrow r^2 = 4r\cos\theta \Rightarrow x^2 + y^2 = 4x$ so it is a circle $(x-2)^2 + y^2 = 4$.)

3. Sketch the polar curve.

$$r = 1 - \cos\theta$$

Ans.