

Name: _____

Section: 205, 206

Write clearly, justify your answers, do not forget your name and your recitation number.

1. Find the (a) velocity and (b) position vectors of a particle that has the given acceleration and the given initial velocity and position.

$$a(t) = 6ti + 12t^2j - 6tk, \quad v(0) = i - j + 3k, \quad r(0) = 0.$$

Ans.

$$v(t) = 3t^2i + 4t^3j - 3t^2k + C$$

By using the initial condition at $t = 0$: $v(0) = C = i - j + 3k$

$$\Rightarrow v(t) = (3t^2 + 1)i + (4t^3 - 1)j + (3 - 3t^2)k.$$

$$r(t) = (t^3 + t)i + (t^4 - t)j + (3t - t^3)k + D$$

At $t = 0$: $r(0) = D = 0$

$$\Rightarrow r(t) = (t^3 + t)i + (t^4 - t)j + (3t - t^3)k.$$

2. A curve is given by

$$x = \frac{1}{2}t^2, \quad y = -t, \quad z = t^3.$$

(a) Find the equation of the normal plane at $(1/2, -1, 1)$.

(Note that the plane determined by the normal and binomial vectors N and B at a point P on a curve C is called the **normal plane** of C at P).

Ans. By the construction of the normal plane, $T(t)$ is the normal vector of it. Since we only need the direction of $T(t)$, we can use $r'(t)$ as the normal vector of the normal plane. For $r(t) = \langle \frac{1}{2}t^2, -t, t^3 \rangle$, $r'(t) = \langle t, -1, 3t^2 \rangle$. At $t = 1$,

$$r'(1) = \langle 1, -1, 3 \rangle.$$

Hence the normal plane equation is

$$(x - \frac{1}{2}) - (y + 1) + 3(z - 1) = 0$$

or

$$x - y + 3z = \frac{9}{2}.$$

(b) At what point on the given curve is the normal plane parallel to the plane

$$2x - 2y + 6z = 3$$

Ans. The normal vector $r'(t) = \langle t, -1, 3t^2 \rangle$ should be parallel to the normal vector of the given plane $\langle 2, -2, 6 \rangle$. In other words, they should be a constant multiple of each other. By comparing the y -components, we want to find t value satisfying

$$2 \langle t, -1, 3t^2 \rangle = \langle 2t, -2, 6t^2 \rangle = \langle 2, -2, 6 \rangle$$

This equality occurs when $t = 1$. So the point on the curve at $t = 1$ is

$$(1/2, -1, 1)$$

3. Find the curvature $\kappa(x)$ of the curve $y = x^4$ at the point $(1, 1)$. Note that

$$\kappa(x) = \frac{|f''(x)|}{[1 + f'(x)^2]^{3/2}}.$$

Ans. $f'(x) = 4x^3$, $f''(x) = 12x^2$. Thus, at $t = 1$,

$$\kappa(1) = \frac{12}{(1 + 16)^{3/2}} = \frac{12}{17\sqrt{17}}.$$