## Quiz 6 Solution for Math104, 2008

**1.** Consider the power series

$$\sum_{n=1}^{\infty} \frac{(-1)^n (x-2)^n}{n3^n}$$

Find the radius of convergence and the interval of convergence for this series.

**Ans.** Use the Ratio test to find the radius of convergence first :

$$\lim_{n\to\infty} \left| \frac{a_{n+1}}{a_n} \right| = \lim_{n\to\infty} \left| \frac{(-1)^{n+1}(x-2)^{n+1}}{(n+1)3^{n+1}} \cdot \frac{n3^n}{(-1)^n (x-2)^n} \right|$$

$$= \lim_{n\to\infty} \frac{|x-2|}{3} \cdot \frac{n}{n+1} = \frac{|x-2|}{3} < 1$$

$$\Rightarrow |x-2| < 3$$

So we get the radius of convergence $R = 3$ and get the interval $-1 < x < 5$.
Check the convergence at the end points :
<u>$x = -1$</u>

$$\sum_{n=1}^{\infty} \frac{(-1)^n(-1-2)^n}{n3^n} = \sum_{n=1}^{\infty} \frac{(-1)^n(-3)^n}{n3^n} = \sum_{n=1}^{\infty} \frac{(-1)^{2n}3^n}{n3^n} = \sum_{n=1}^{\infty} \frac{1}{n}$$

This is the harmonic series which is divergent.

<u>$x = 5$</u>

$$\sum_{n=1}^{\infty} \frac{(-1)^n(5-2)^n}{n3^n} = \sum_{n=1}^{\infty} \frac{(-1)^n 3^n}{n3^n} = \sum_{n=1}^{\infty} \frac{(-1)^n}{n}$$

This is the alternating harmonic series which is convergent.

Thus, the interval of convergence of the given series is

$$-1 < x \leq 5$$

**2.** Find the Taylor series for $f(x) = \cos x$ at the given value $a = \pi$. (Note that the Taylor series of $f(x)$ at $a$ is $f(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(a)}{n!}(x-a)^n$.)

**Ans.**

| $n$ | $f^{(n)}(x)$ | $f^{(n)}(\pi)$ |
|---|---|---|
| 0 | $\cos x$ | $-1$ |
| 1 | $-\sin x$ | 0 |
| 2 | $-\cos x$ | 1 |
| 3 | $\sin x$ | 0 |
| 4 | $\cos x$ | $-1$ |
| ... | ... | ... |

$$\cos x = -1 + \frac{(x-\pi)^2}{2!} - \frac{(x-\pi)^4}{4!} + \frac{(x-\pi)^6}{6!} + \cdots = \sum_{n=0}^{\infty} (-1)^{n+1} \frac{(x-\pi)^{2n}}{(2n)!}$$

**3.** Find the sum of the series. (Note that $e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!}$.)

$$1 - \ln 2 + \frac{(\ln 2)^2}{2!} - \frac{(\ln 2)^3}{3!} + \cdots$$

**Ans.**

$$1 - \ln 2 + \frac{(\ln 2)^2}{2!} - \frac{(\ln 2)^3}{3!} + \cdots = \sum_{n=0}^{\infty} \frac{(-\ln 2)^n}{n!} = e^{-\ln 2} = 2^{-\ln e} = 2^{-1}$$

$$= \frac{1}{2}$$