MATH 114, 2009 Quiz # 6 Solution Tuesday

Name:______________________ Section: 205, 206

1. (1) Find the following limit if it exists or explain why it doesn't exist.

$$\lim_{(x,y)\to(0,0)} \frac{x^2y^2}{x^4+y^5}$$

Ans. The limit does not exist and we show this by finding two different paths having two different limit values :
Along $x = 0$:

$$\lim_{(0,y)\to(0,0)} \frac{0}{y^5} = 0, \quad \text{for } y \neq 0$$

Along $x = y$:

$$\lim_{(x,x)\to(0,0)} \frac{x^4}{x^4+x^5} = \lim_{(x,x)\to(0,0)} \frac{1}{1+x} = 1$$

Since the limit value along the path $x = 0$ is 0 and the limit value along $x = y$ is 1 and they are not equal, the limit value does not exist.

(2). $f(x) = \frac{\sin(x^2)}{x^2}$ is a well-defined function except at $x = 0$, since the denominator is zero when $x = 0$. Is it possible to assign a value to $f(x = 0)$ such that it's a continuous function anywhere on the real line (i.e., continuous for any x)?

Ans. Note that the definition of continuity at a point $x = a$ requires the following three things :

1. $f(a)$ is defined
2. $\lim_{x\to a} f(x)$ exists
3. $\lim_{x\to a} = f(a)$

So, all we need to do is defining the function value $f(x = 0)$ at $x = 0$ by the limit value $\lim_{x\to 0} f(x)$ so that all three conditions can be satisfied.

$$\lim_{x\to 0} f(x) = \lim_{x\to 0} \frac{\sin(x^2)}{x^2} = \lim_{x\to 0} \frac{2x\cos(x^2)}{2x} = \lim_{x\to 0} \cos(x^2) = 1.$$

(Note that we used the L'Hopital's rule in the second step since it was 0/0 case). Hence, if we define

$$f(x = 0) = 1$$

then $f(x)$ is continuous everywhere.

2. $f(x,y) = e^{x^2}\sin\sqrt{xy}$. Find the following partial derivatives : $f_x \quad f_y \quad f_{yx}$

Ans.

$$\begin{aligned}
f_x(x,y) &= 2xe^{x^2}\sin\sqrt{xy} + \frac{1}{2}\sqrt{\frac{y}{x}}e^{x^2}\cos\sqrt{xy} \\
f_y(x,y) &= \frac{1}{2}\sqrt{\frac{x}{y}}e^{x^2}\cos\sqrt{xy} \\
f_{yx}(x,y) &= \frac{1}{4\sqrt{xy}}e^{x^2}\cos\sqrt{xy} + x\sqrt{\frac{x}{y}}e^{x^2}\cos\sqrt{xy} - \frac{1}{4}e^{x^2}\sin\sqrt{xy}
\end{aligned}$$